

Question 1

Consider the autonomous equation

$$y' = y(1 - y)(y - 3), \quad y(0) = a \in \mathbb{R}.$$

An equilibrium is a constant solution. Call it stable if sufficiently small initial perturbations stay as small as prescribed for all future time, and asymptotically stable if it is also locally attracting. A basin of attraction is the set of initial values whose forward solutions exist for all $t \geq 0$ and tend to that equilibrium.

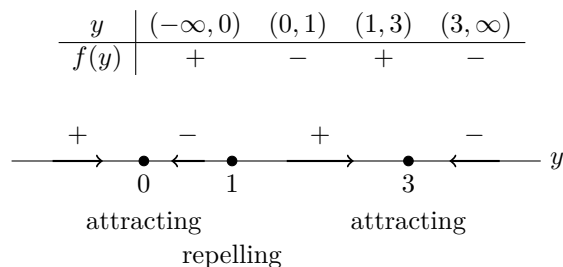
Tasks

1. Find all equilibria and construct a phase line, showing the sign of y' on every interval between them.
2. Classify the equilibria as asymptotically stable or unstable, explaining the role of the arrows.
3. Determine the forward limit for every real initial value a and the exact basins of the attracting equilibria.
4. Justify global forward existence and explain why a nonconstant solution cannot cross an equilibrium. Do not rely on an explicit solution formula.

Question 1 – Solution

Strategy. Use sign intervals and uniqueness to trap solutions; bounded monotone trajectories reveal their limits.

Step 1: Find the zeros and arrows. The only zeros of $f(y) = y(1-y)(y-3)$ are 0, 1, 3. Its signs are



Step 2: Classify the equilibria. Arrows point toward 0 and 3 on both sides, so both are asymptotically stable. The arrows on either side of 1 point away, so 1 is unstable. Nearby solutions remain between their initial value and the attracting zero, which supplies stability as well as attraction.

Step 3: Determine every forward limit. The classification is

$$\lim_{t \rightarrow \infty} y(t) = \begin{cases} 0, & a < 1, \\ 1, & a = 1, \\ 3, & a > 1. \end{cases}$$

In particular, the basins are $(-\infty, 1)$ for 0 and $(1, \infty)$ for 3. These include the attracting equilibria themselves. The value $a = 1$ stays exactly at the unstable equilibrium.

Step 4: Justify the global conclusions. Since f is a polynomial, local solutions are unique. A nonconstant solution cannot meet an equilibrium at a finite time: local uniqueness there, in both time directions, would force the two solutions to agree.

Every trajectory is trapped in a bounded interval between its initial value and adjacent equilibria. A smooth right-hand side permits continuation while the solution remains bounded, so all forward solutions are global. Each nonconstant trajectory is monotone and bounded. If its limit L had $f(L) \neq 0$, continuity would keep its speed bounded away from zero near L , contradicting convergence. The accessible zero in its direction of motion is therefore its limit.

Question 2

Consider

$$y' = -(y + 1)(y - 2)^2.$$

Call an equilibrium semistable if it attracts from one side and repels from the other; such an equilibrium is not stable under unrestricted perturbations from both sides.

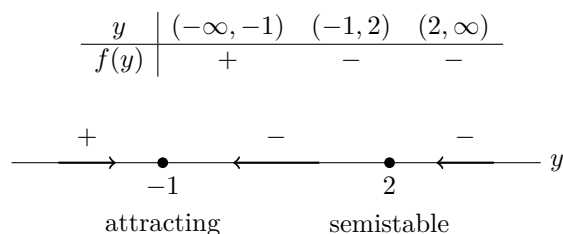
Tasks

1. Find the equilibria and construct a complete phase line, including signs on both sides of a repeated zero.
2. Classify each equilibrium and specify the attracting side of any semistable equilibrium.
3. Find the forward limits for initial values $y(0) = -2$, 1 , and 3 , with a justification of forward existence.
4. Evaluate f' at each equilibrium. Explain why a zero derivative is not itself a stability classification and why the repeated factor does not reverse the sign of f .

Question 2 – Solution

Strategy. Preserve the sign across a squared factor; use one-sided attraction and repulsion when the derivative test is inconclusive.

Step 1: Build the sign chart. The equilibria are -1 and 2 . Away from them, $(y - 2)^2 > 0$, so the sign of f is the opposite of the sign of $y + 1$:



Step 2: Classify with the sides stated. The equilibrium -1 is asymptotically stable. At 2 , solutions above it decrease toward it, while those just below decrease away toward -1 . Hence

2 is semistable: attracting from above, repelling from below.

It is unstable in the two-sided stability sense: however close an initial value is below 2 , its trajectory eventually leaves, for example, the interval $(1, 3)$.

Step 3: Track the three initial values. Uniqueness prevents crossing either equilibrium. The solution starting at -2 increases toward -1 ; that starting at 1 decreases toward -1 ; that starting at 3 decreases toward 2 . Each remains in a bounded closed interval, so the smooth equation has a global forward solution in each case. Monotone convergence and the nonzero-speed contradiction identify the stated limits.

Step 4: Interpret the derivative test. Differentiating gives $f'(y) = -(y - 2)^2 - 2(y + 1)(y - 2)$, so

$$f'(-1) = -9 < 0, \quad f'(2) = 0.$$

The negative derivative confirms attraction at -1 . At 2 , the linearized equation contains no nonzero first-order term and makes no decision. The sign chart supplies the missing information. A squared factor is positive on both sides of its zero, so it touches zero without a sign reversal.

Question 3

Two proposed relaxation laws are

$$\text{A: } y' = -y^3, \quad \text{B: } y' = y^3.$$

Both have an equilibrium at 0 and derivative of the right-hand side equal to zero there.

Tasks

1. Classify 0 for each law using the sign of the right-hand side. Explain why their identical derivative-test data do not imply identical stability.
2. Solve both IVPs with $y(0) = a$, including $a = 0$, and specify their maximal forward time ranges.
3. For law A, prove stability directly from the solution and determine its large-time decay for $a \neq 0$.
4. For law B, use the solution to prove instability and find the blow-up time for $a \neq 0$. Distinguish leaving a small neighborhood from blowing up.

Question 3 – Solution

Strategy. When the linear term vanishes, retain the nonlinear term. Separation gives both a stability proof and the actual time scales.

Step 1: Classify from signs. For A, $-y^3$ is positive below 0 and negative above, so trajectories move toward 0. For B the signs reverse, so trajectories move away. Thus 0 is asymptotically stable for A and unstable for B. The shared value $f'(0) = 0$ is an inconclusive test, not evidence of neutral stability.

Step 2: Solve with the initial sign preserved. For $a \neq 0$, differentiating y^{-2} gives 2 for A and -2 for B. Applying the initial value yields

$$y_A(t) = \frac{a}{\sqrt{1+2a^2t}}, \quad t \geq 0,$$

$$y_B(t) = \frac{a}{\sqrt{1-2a^2t}}, \quad 0 \leq t < \frac{1}{2a^2}.$$

The full maximal intervals containing 0 are $(-1/(2a^2), \infty)$ and $(-\infty, 1/(2a^2))$, respectively. Direct differentiation verifies the equations. If $a = 0$, the unique solution of either smooth equation is zero for all real time.

Step 3: Prove stability and attraction for A. For $t \geq 0$, $|y_A(t)| \leq |a|$. Given $\varepsilon > 0$, choosing $\delta = \varepsilon$ ensures $|a| < \delta$ implies $|y_A(t)| < \varepsilon$ for all future time. Also $y_A(t) \rightarrow 0$ for every a , so the attraction is global. More precisely,

$$\sqrt{2t}|y_A(t)| \rightarrow 1 \quad (a \neq 0).$$

The decay is algebraic rather than exponential.

Step 4: Prove instability for B. Choose any fixed $\varepsilon > 0$ and any nonzero $|a| < \varepsilon$. The magnitude first reaches ε at

$$t_\varepsilon = \frac{1}{2a^2} - \frac{1}{2\varepsilon^2} > 0,$$

then exceeds it. Arbitrarily small nonzero perturbations therefore leave that neighborhood. Blow-up occurs later, at $1/(2a^2)$, with the sign of a . Instability only requires departure from a prescribed neighborhood; it does not require an infinite solution value.

Question 4

Construct a real polynomial autonomous equation $y' = f(y)$ with exactly the equilibria $-2, 0, 3$ and these properties:

- -2 is asymptotically stable;
- 0 attracts from above and repels from below;
- 3 is unstable;
- $f(1) = -6$.

Tasks

1. Determine the smallest possible degree, using the sign changes required at the three zeros.
2. Construct the polynomial of that degree and prove uniqueness among polynomials of that degree satisfying all the requirements.
3. Draw its phase line and verify each requested stability property.
4. Find the forward limits for $y(0) = -1$ and $y(0) = 1$. Explain why knowing the equilibrium locations alone would not determine these limits.

Question 4 – Solution

Strategy. Translate attraction and repulsion into signs, then into the parity of each root's multiplicity.

Step 1: Establish the minimum degree. At a two-sided attracting or repelling equilibrium, the sign must reverse, requiring an odd root multiplicity. At the specified semistable zero, the sign must be negative on both sides, requiring even multiplicity. The smallest multiplicities at $-2, 0, 3$ are therefore $1, 2, 1$, so the degree is at least 4.

Step 2: Determine the coefficient. Every degree-four candidate must have the form

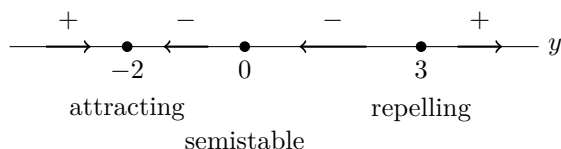
$$f(y) = K(y + 2)y^2(y - 3), \quad K \neq 0.$$

At $y = 1$ this gives $f(1) = -6K$. The prescribed rate forces $K = 1$, hence

$$y' = (y + 2)y^2(y - 3).$$

This has exactly the three requested real zeros. The forced multiplicities exhaust degree four, and the rate condition fixes the only remaining coefficient, proving uniqueness at the minimum degree.

Step 3: Verify the phase line. The signs are $+, -, -, +$ on the four successive intervals cut by $-2, 0, 3$.



Thus arrows approach -2 from both sides; near 0 they point left on both sides, attracting from above and repelling from below; and they point away from 3 on both sides. These are precisely the requested behaviors.

Step 4: Infer the selected limits. Starting at -1 , the solution decreases within $(-2, 0)$ and tends to $\boxed{-2}$. Starting at 1 , it decreases within $(0, 3)$ and tends to $\boxed{0}$. Smoothness and bounded trapping ensure global forward existence for both, and uniqueness prevents crossing the bounding equilibria.

Equilibrium locations only identify zeros. Changing root multiplicities or the overall sign can change the arrows and therefore the limiting behavior, even when the zero set is unchanged. The stability and rate requirements supply information that the locations alone do not.

Question 5

Consider the one-parameter family

$$y' = \mu - y^2, \quad \mu \in \mathbb{R}.$$

Tasks

1. Find and classify all equilibria for $\mu < 0$, $\mu = 0$, and $\mu > 0$. State the one-sided behavior at any semistable case.
2. Draw the equilibrium branches in the (μ, y) -plane, distinguishing attracting and repelling branches and marking the exceptional parameter value.
3. Solve the IVP $y(0) = 0$ for each parameter regime and determine its forward behavior and maximal forward time range.
4. Explain how the number and stability of equilibria change at $\mu = 0$, and why the derivative test alone does not settle that parameter value.

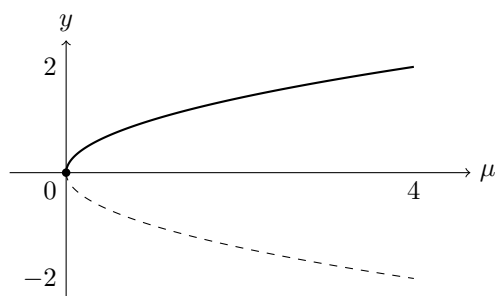
Question 5 – Solution

Strategy. Compare a horizontal parameter with y^2 , and treat the double zero separately from the two simple roots.

Step 1: Classify the parameter regimes. If $\mu < 0$, there are no equilibria and $y' < 0$ everywhere. If $\mu = 0$, the only equilibrium is 0: $-y^2 < 0$ on both sides, so it attracts from above and repels from below. If $\mu > 0$, the equilibria are

$$y = -\sqrt{\mu} \text{ (unstable),} \quad y = \sqrt{\mu} \text{ (asymptotically stable)}.$$

Indeed $\mu - y^2$ is positive between the roots and negative outside them. The derivative $f_y = -2y$ confirms the classifications at the simple roots.



Solid: attracting; dashed: repelling; origin: semistable

Step 2: Solve the selected IVP. Separation and $y(0) = 0$ give the following cases, with $a > 0$ as indicated:

parameter	$y(t)$	forward time range
$\mu > 0, a = \sqrt{\mu}$	$a(1 - e^{-2at})/(1 + e^{-2at})$	$0 \leq t < \infty$
$\mu = 0$	0	$0 \leq t < \infty$
$\mu < 0, a = \sqrt{-\mu}$	$-a \tan(at)$	$0 \leq t < \pi/(2a)$

For $\mu > 0$, integration on $-a < y < a$ gives $\ln((a + y)/(a - y)) = 2at$. For the last case, integration gives $\arctan(y/a) = -at$ on the initial branch. Differentiating the displayed expressions verifies $y' = \mu - y^2$ and their zero initial values.

Step 3: Describe the future. For $\mu > 0$, the selected solution increases to $\sqrt{\mu}$; at $\mu = 0$ it stays at zero; for $\mu < 0$ it decreases to $-\infty$ as $t \uparrow \pi/(2a)$, so no finite continuous extension exists.

Step 4: Interpret the branch merger. As μ decreases to zero, the attracting and repelling equilibria merge into a semistable double zero; for negative μ neither remains. At the merger $f_y(0) = 0$, so the derivative test is inconclusive. The sign of $-y^2$ supplies the one-sided classification.

Question 6

For a fixed real constant L , compare the two equations on $t \geq 0$:

$$\text{A: } y' = -\frac{y-L}{(1+t)^2}, \quad \text{B: } y' = -\frac{y-L}{1+t}.$$

Their right-hand sides depend on time. Nevertheless, at every finite time each points toward L whenever $y \neq L$. Use stability relative to the initial time $t = 0$.

Tasks

1. Find every constant solution of each equation and explain why these are equilibria despite the explicit time dependence.
2. Solve both IVPs with $y(0) = a$ and verify the initial values and differential equations.
3. Decide whether L is stable and whether it is asymptotically stable for each equation. Prove the claims directly from the solutions.
4. Explain why arrows pointing toward L at every finite time do not guarantee attraction in both models. Relate the distinction to the accumulated coefficient integral.

Question 6 – Solution

Strategy. Measure the displacement from L . Its reduction depends on the total accumulated damping, not just the sign at each instant.

Step 1: Identify the constant solutions. A constant $y = c$ must make the right-hand side zero for every $t \geq 0$. Both coefficients are strictly positive, so the only choice in either equation is $\boxed{c = L}$. Explicit time dependence does not preclude a constant solution.

Step 2: Solve the displacement equations. Let $z = y - L$, so $z(0) = a - L$. For A,

$$\int_0^t \frac{ds}{(1+s)^2} = 1 - \frac{1}{1+t} = \frac{t}{1+t}, \quad \boxed{y_A = L + (a - L)e^{-t/(1+t)}}.$$

For B, the corresponding integral is $\ln(1+t)$, giving

$$\boxed{y_B = L + \frac{a - L}{1 + t}}.$$

Both formulas have value a at zero. Their displacement derivatives are exactly the prescribed negative coefficient times their displacement, verifying the equations. They exist for all $t \geq 0$.

Step 3: Separate stability from attraction. For both models, $|y(t) - L| \leq |a - L|$. Given $\varepsilon > 0$, $\delta = \varepsilon$ proves stability at initial time zero. But

$$\lim_{t \rightarrow \infty} y_A = L + (a - L)e^{-1}, \quad \lim_{t \rightarrow \infty} y_B = L.$$

Thus L is **stable but not asymptotically stable for A**; no other initial value converges to L . For B, L is **globally asymptotically stable**.

Step 4: Explain the limitation of the arrow argument. In a scalar equation $z' = -b(t)z$, the solution is $z(0) \exp(-\int_0^t b(s) ds)$. A has positive but integrable damping, with total integral 1; a fixed fraction of any nonzero initial displacement remains. B has divergent accumulated damping and removes the displacement completely.

For a continuous autonomous equation, a non-equilibrium limiting value would have nonzero limiting speed. Here the coefficient can tend to zero with time, so that autonomous argument does not apply. Instantaneous inward arrows alone are insufficient for a time-dependent equation.

Question 7

Consider the continuous, non-Lipschitz equation

$$y' = 2\sqrt{|y|}, \quad y(0) = 0, \quad t \geq 0.$$

When solutions are not unique, call an equilibrium stable only if every forward solution from every sufficiently close initial value remains within any prescribed neighborhood.

Tasks

1. Find all equilibria and determine the sign of the right-hand side away from them.
2. For each $c \geq 0$, construct a solution that stays at zero through time c and then becomes positive. Verify differentiability at the joining time.
3. Construct a continuously differentiable solution with $y(0) = -1$ that crosses the equilibrium level at $t = 1$, and sketch it.
4. Prove that the equilibrium is unstable under the stated convention. Explain precisely why the usual prohibition against crossing an equilibrium does not apply.

Question 7 – Solution

Strategy. Verify explicit joined solutions instead of invoking a uniqueness theorem whose hypothesis fails.

Step 1: Find the zero and signs. The only equilibrium is $y = 0$. The right-hand side is positive for every $y \neq 0$, so motion is upward on either side. Below zero the direction is toward the equilibrium level, but arrival need not force the solution to stay there.

Step 2: Verify delayed departures. For any $c \geq 0$, define

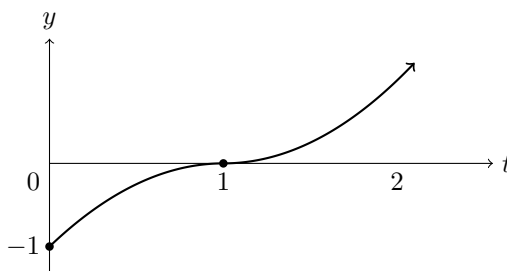
$$y_c(t) = \begin{cases} 0, & 0 \leq t \leq c, \\ (t - c)^2, & t \geq c. \end{cases}$$

Before c both sides of the equation vanish; afterward $y'_c = 2(t - c) = 2\sqrt{|y_c|}$. At c , both values and both one-sided derivatives are zero. The difference quotient also gives derivative zero, so the joined solution is continuously differentiable. The identically zero function is another solution.

Step 3: Cross from below. A crossing solution is

$$y(t) = \begin{cases} -(1 - t)^2, & 0 \leq t \leq 1, \\ (t - 1)^2, & t \geq 1. \end{cases}$$

Before 1, $y' = 2(1 - t) = 2\sqrt{|y|}$; afterward the same check as above applies. Both derivatives at the join are zero, and $y(0) = -1$.



A classical solution crosses $y = 0$ at $t = 1$

Step 4: Diagnose instability and failed uniqueness. For any $\delta > 0$, initial value 0 is within δ of the equilibrium, yet the solution $y_0(t) = t^2$ leaves the fixed neighborhood $(-1, 1)$ after $t = 1$. This violates the stated requirement for every forward solution, so the equilibrium is unstable.

Local Lipschitz regularity fails since $|f(y) - f(0)|/|y| = 2/\sqrt{|y|}$ is unbounded near zero. The crossing and constant solutions agree at $(1, 0)$ but have different continuations. The usual noncrossing proof requires local uniqueness; continuity alone does not provide it.

Question 8

Consider the autonomous equation with its stated domain:

$$y' = \frac{y-1}{y+2}, \quad y \neq -2.$$

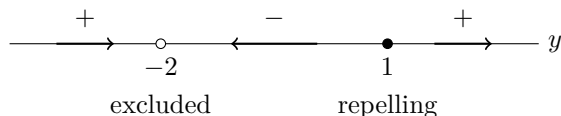
Tasks

1. Identify all equilibria and construct a phase line that distinguishes an excluded level from a zero of the right-hand side.
2. Classify each equilibrium and determine the direction of motion in every component cut by the equilibrium and the excluded level.
3. For $y(0) = 0$, derive an implicit solution relation and find the exact maximal forward time range.
4. Determine the limiting value and slope at its finite endpoint. Explain why approaching a finite level does not make that level an attracting equilibrium and why multiplying by $y + 2$ does not remove the obstruction for this original equation.

Question 8 – Solution

Strategy. A denominator zero splits the domain but is not an equilibrium. Test whether the selected trajectory reaches that boundary in finite time.

Step 1: Separate zeros from exclusions. The only equilibrium is $y = 1$. The value $y = -2$ is outside the equation's domain. The signs of f are positive on $(-\infty, -2)$, negative on $(-2, 1)$, and positive on $(1, \infty)$.



Thus 1 is unstable: arrows point away on both sides. Also $f'(y) = 3/(y+2)^2$, so $f'(1) = 1/3 > 0$ confirms the classification.

Step 2: Integrate the selected branch. Starting at 0, the solution decreases within $(-2, 1)$. Separation gives

$$dt = \frac{y+2}{y-1} dy, \quad t = y + 3 \ln(1-y).$$

The constant is zero from $y(0) = 0$. On $-2 < y \leq 0$, the derivative of the right side with respect to y is $(y+2)/(y-1) < 0$. It is therefore invertible along this branch, and implicit differentiation verifies the original equation.

Step 3: Determine the endpoint. As $y \downarrow -2$, the time tends to

$$T = -2 + 3 \ln 3 \approx 1.296, \quad 0 \leq t < T.$$

This is the maximal forward range: the inverse exists until T , but at T its limiting value is excluded. Within the branch,

$$y(t) \rightarrow -2^+, \quad y'(t) = \frac{y-1}{y+2} \rightarrow -\infty.$$

Step 4: Interpret the domain boundary. An attracting equilibrium must itself define a constant solution of the given equation. Here -2 does not, and the selected solution reaches the boundary in finite time with unbounded slope rather than converging to a valid equilibrium as $t \rightarrow \infty$.

Multiplication gives $(y+2)y' = y-1$, equivalent to the original only when $y \neq -2$. At $y = -2$ it would demand $0 = -3$ for any finite derivative. Thus even the multiplied relation supplies no differentiable continuation through this boundary.

Question 9

Let

$$V(y) = \frac{1}{4}(y^2 - 1)^2, \quad y' = -V'(y).$$

Think of V as an energy function; no physical units are assumed.

Tasks

1. Derive the autonomous equation explicitly, find its equilibria, and classify them using the signs of the right-hand side.
2. Compute $dV(y(t))/dt$ along a solution. Identify exactly when this derivative vanishes.
3. Sketch V and relate its local minima and maximum to equilibrium stability. Explain why nonincreasing energy alone does not imply convergence to a global minimum for every initial value.
4. Determine the forward limit for every real initial value and justify global forward existence. State the basins of the attracting equilibria.

Question 9 – Solution

Strategy. Combine the energy identity with the phase line; energy decrease is informative but does not by itself choose a limiting equilibrium.

Step 1: Derive the flow and classify its zeros. Since $V'(y) = y(y^2 - 1)$,

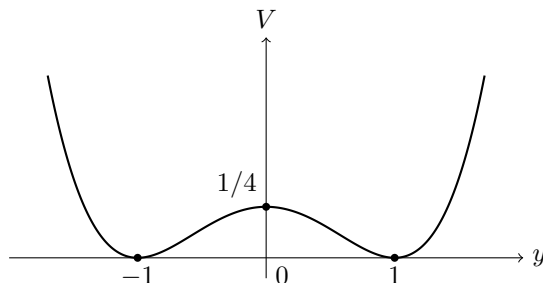
$$y' = y - y^3 = y(1 - y^2).$$

Its zeros are $-1, 0, 1$. Its signs on the successive intervals are $+, -, +, -$, so -1 and 1 are asymptotically stable and 0 is unstable.

Step 2: Differentiate the energy. The chain rule gives

$$\frac{dV}{dt} = V'(y)y' = -[V'(y)]^2 = -(y - y^3)^2 \leq 0.$$

The derivative vanishes exactly at $y = -1, 0, 1$. On any nonconstant trajectory it is strictly negative, since uniqueness prevents meeting an equilibrium at a finite time.



Energy minima: ± 1 ; local maximum: 0

Step 3: Interpret the energy landscape. The points ± 1 are minima with $V = 0$; 0 is a local maximum with $V = 1/4$. Since the flow is down the energy gradient, the minima attract and the maximum repels. Nevertheless, the initial value $y(0) = 0$ stays at that maximum forever, with constant energy. Nonincreasing energy does not force every solution to reach a global minimum, nor distinguish which minimum will be selected.

Step 4: Determine the basins and future limits. The decreasing-energy bound confines a trajectory to $\{y : V(y) \leq V(y(0))\}$, a bounded set because $V(y) \rightarrow \infty$ as $|y| \rightarrow \infty$. Smoothness then ensures global forward continuation. Phase-line trapping gives bounded monotonic motion for every nonconstant solution. Its limit must be a zero of $y - y^3$, and it cannot cross zero. Therefore

$$\lim_{t \rightarrow \infty} y(t) = \begin{cases} -1, & y(0) < 0, \\ 0, & y(0) = 0, \\ 1, & y(0) > 0. \end{cases}$$

The basins are $(-\infty, 0)$ for -1 and $(0, \infty)$ for 1 .

Question 10

Compare two autonomous equations on the real line:

$$\text{A: } y' = -y, \quad \text{B: } y' = -\frac{y}{1+y^2}.$$

For the quantitative comparison, use the same initial value $y(0) = a > 0$.

Tasks

1. Compare the equilibrium sets, phase-line arrows and derivatives of the right-hand sides at the equilibria. Prove the stability classification for both equations.
2. Find the solution of A and an implicit relation for B. Justify global forward existence and convergence for B.
3. Derive the time required to decrease from a to $a/2$ for each equation. Explain how the difference depends on a .
4. For $a = 2$, sketch both trajectories and determine which is larger for every $t > 0$. Explain what identical equilibrium and local derivative data fail to specify.

Question 10 – Solution

Strategy. Multiplication by a positive state-dependent factor preserves the direction of motion, but changes the time spent along the same phase-line path.

Step 1: Compare equilibrium information. Both equations have only the equilibrium 0, with positive velocity below zero and negative velocity above. Both right-hand-side derivatives at zero equal -1 . In each model, a solution is trapped between its initial value and zero, which proves stability. Smoothness, boundedness and monotonicity give global forward existence and convergence to the only accessible zero. Thus both equilibria are globally asymptotically stable.

Step 2: Solve the selected branches. For A, $y_A = ae^{-t}$. For B with $y > 0$, separation gives

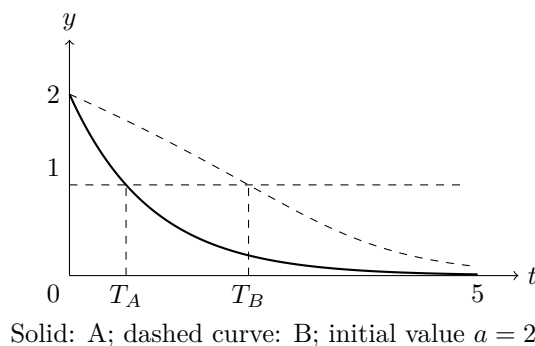
$$dt = -(1/y + y)dy, \quad t = \ln(a/y) + \frac{a^2 - y^2}{2}.$$

The right side decreases strictly with y on $(0, a]$, is zero at a , and tends to infinity as $y \downarrow 0$. It therefore defines a unique positive solution for every $t \geq 0$. Implicit differentiation recovers B and confirms that zero is approached only in infinite time.

Step 3: Compare halving times. Setting $y = a/2$ yields

$$T_A = \ln 2, \quad T_B = \ln 2 + \frac{3a^2}{8}.$$

The extra time in B grows quadratically with the initial amplitude. It tends to zero as $a \downarrow 0$, consistent with the identical local derivatives, but can be arbitrarily large for large a .



Step 4: Order the trajectories. For $t > 0$, the B solution satisfies $0 < y_B < a$, so its implicit equation gives $t > \ln(a/y_B)$. Hence $y_B > ae^{-t} = y_A$. The graph uses $a = 2$, for which $T_B = \ln 2 + 3/2$.

The factor $1/(1 + y^2)$ is positive and equals 1 at zero, preserving phase directions and the local linear rate. Away from zero it slows the motion. Equilibria and local stability data alone do not specify finite-amplitude travel times or entire solution curves.