

Question 3

Work over the real numbers. Use substitution or elimination, keeping track of conditions under which an operation preserves all solutions. Check candidates in the original equations.

A system begins with an equation whose x coefficient is zero:

$$2y - z = 1, \quad x - y + 2z = 3, \quad 2x + y + z = 6 + \delta,$$

where δ is a real perturbation of the last measurement.

Tasks

1. Solve the system for $\delta = 0$ using valid elimination. Explain why the initial zero coefficient is not evidence of nonuniqueness.
2. Find the exact solution for arbitrary δ , keeping fractions exact.
3. Verify the perturbed solution in all original equations, and decide whether any δ makes the system inconsistent or nonunique.
4. Compute $x + 3y$ and recover δ from an exact measurement of z . Explain why measuring $x + 3y$ alone cannot detect the perturbation.

Question 3 – Solution

Strategy. Choose an equation with a nonzero coefficient for elimination. An inconvenient ordering does not change the information in a system.

Step 1: Eliminate without dividing by zero. The second equation gives $x = 3 + y - 2z$. Substituting into the third gives $3y - 3z = \delta$, or $y - z = \delta/3$. Together with $2y - z = 1$, this determines y, z , then x . For $\delta = 0$ the result is $(x, y, z) = (2, 1, 1)$. Swapping the first two equations would also provide an immediate nonzero x coefficient; neither approach requires a zero division.

Step 2: Solve for the perturbation exactly. Subtract $y - z = \delta/3$ from $2y - z = 1$ to obtain $y = 1 - \delta/3$. It follows that

$$x = 2 + \delta, \quad y = 1 - \frac{\delta}{3}, \quad z = 1 - \frac{2\delta}{3}.$$

Only the nonzero constants 1 and 3 were used as divisors, so this calculation applies to every real δ .

Step 3: Check the original measurements and uniqueness. Substitution gives

$$\begin{aligned} 2y - z &= 2 - 2\delta/3 - 1 + 2\delta/3 = 1, \\ x - y + 2z &= 2 + \delta - 1 + \delta/3 + 2 - 4\delta/3 = 3, \\ 2x + y + z &= 4 + 2\delta + 1 - \delta/3 + 1 - 2\delta/3 = 6 + \delta. \end{aligned}$$

Every solution was forced by reversible substitutions and nonzero divisions. Thus exactly one solution exists for every real δ .

Step 4: Identify a blind measurement. The combination $x + 3y = 2 + \delta + 3 - \delta = 5$ is independent of δ . In contrast, z determines $\delta = \frac{3}{2}(1 - z)$ uniquely. The first two equations fix certain combinations even when the last datum changes; a measurement that stays constant along that change cannot reveal it.