

Question 1

Consider

$$y' = t - y, \quad y(0) = 1.$$

For a step h , explicit Euler uses $t_{n+1} = t_n + h$ and $Y_{n+1} = Y_n + hf(t_n, Y_n)$, where Y_n approximates $y(t_n)$. Use exact arithmetic in intermediate steps unless rounding is explicitly requested.

Tasks

1. Starting at $t_0 = 0$, use $h = 1/4$ to compute every Euler value through $t = 1$. Include the slope used on each step.
2. Find the exact solution and the signed endpoint error $Y_4 - y(1)$, reporting its numerical value to six decimal places.
3. Draw the Euler polygon and the exact solution on $[0, 1]$. Explain what the slope of each straight segment represents.
4. A program obtains $Y_1 = 0.8125$ by evaluating $t - y$ at (t_1, Y_0) . Identify the error and show the correct first update. Is that program implementing the stated Euler formula?

Question 1 – Solution

Strategy. Evaluate each slope at the current numerical point, then move both time and the approximation to the next node.

Step 1: Compute the four updates. The recurrence is $Y_{n+1} = Y_n + (t_n - Y_n)/4$. Its exact values are

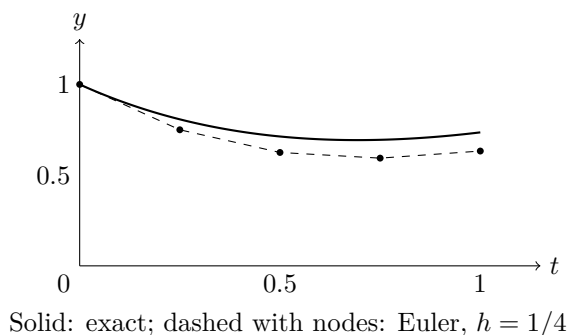
n	t_n	Y_n	$f(t_n, Y_n)$
0	0	1	-1
1	1/4	3/4	-1/2
2	1/2	5/8	-1/8
3	3/4	19/32	5/32
4	1	81/128	not needed

Thus $Y_4 = 81/128 = 0.6328125$.

Step 2: Check against the exact IVP. Solving $y' + y = t$ with integrating factor e^t gives

$$y = t - 1 + 2e^{-t}, \quad Y_4 - y(1) = \frac{81}{128} - \frac{2}{e} \approx -0.102946.$$

The exact formula has $y(0) = 1$ and derivative $1 - 2e^{-t} = t - y$, so it is the correct comparison solution.



Step 3: Interpret the polygon. On $[t_n, t_{n+1}]$, the Euler segment has slope $f(t_n, Y_n)$. It is tangent to the solution through the numerical point (t_n, Y_n) , which generally differs from the original exact solution after the first step. The polygon is continuous but usually has corners at its nodes.

Step 4: Diagnose the time index. The correct first update is $1 + (1/4)(0 - 1) = 0.75$. The program instead uses $1 + (1/4)(1/4 - 1) = 0.8125$, mixing the new time with the old state. It is not the stated explicit Euler method, which requires both arguments from the same current node.