

Question 2

Consider

$$y' = -(y + 1)(y - 2)^2.$$

Call an equilibrium semistable if it attracts from one side and repels from the other; such an equilibrium is not stable under unrestricted perturbations from both sides.

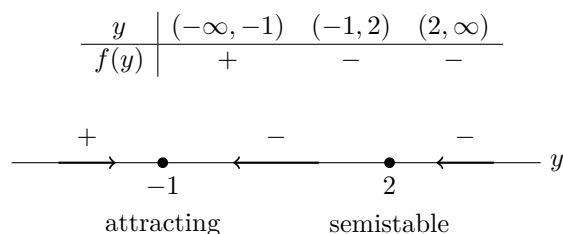
Tasks

1. Find the equilibria and construct a complete phase line, including signs on both sides of a repeated zero.
2. Classify each equilibrium and specify the attracting side of any semistable equilibrium.
3. Find the forward limits for initial values $y(0) = -2$, 1 , and 3 , with a justification of forward existence.
4. Evaluate f' at each equilibrium. Explain why a zero derivative is not itself a stability classification and why the repeated factor does not reverse the sign of f .

Question 2 – Solution

Strategy. Preserve the sign across a squared factor; use one-sided attraction and repulsion when the derivative test is inconclusive.

Step 1: Build the sign chart. The equilibria are -1 and 2 . Away from them, $(y - 2)^2 > 0$, so the sign of f is the opposite of the sign of $y + 1$:



Step 2: Classify with the sides stated. The equilibrium -1 is asymptotically stable. At 2 , solutions above it decrease toward it, while those just below decrease away toward -1 . Hence

2 is semistable: attracting from above, repelling from below.

It is unstable in the two-sided stability sense: however close an initial value is below 2 , its trajectory eventually leaves, for example, the interval $(1, 3)$.

Step 3: Track the three initial values. Uniqueness prevents crossing either equilibrium. The solution starting at -2 increases toward -1 ; that starting at 1 decreases toward -1 ; that starting at 3 decreases toward 2 . Each remains in a bounded closed interval, so the smooth equation has a global forward solution in each case. Monotone convergence and the nonzero-speed contradiction identify the stated limits.

Step 4: Interpret the derivative test. Differentiating gives $f'(y) = -(y - 2)^2 - 2(y + 1)(y - 2)$, so

$$f'(-1) = -9 < 0, \quad f'(2) = 0.$$

The negative derivative confirms attraction at -1 . At 2 , the linearized equation contains no nonzero first-order term and makes no decision. The sign chart supplies the missing information. A squared factor is positive on both sides of its zero, so it touches zero without a sign reversal.