

Question 1

A tank initially holds 100 L of well-mixed solution containing 10 kg of salt. Solution containing 0.20 kg/L enters at 3 L/min, and the mixture leaves at 1 L/min. The tank has capacity 200 L. Assume instantaneous mixing, additive liquid volumes, and no salt precipitation or other losses. Let t be minutes after the flows begin.

Tasks

1. Derive initial-value models for the liquid volume $V(t)$ and salt mass $S(t)$, including units for all rate terms and the operating time before overflow.
2. Derive and solve an equation for the concentration $c(t) = S(t)/V(t)$. Explain why c' is not simply S'/V .
3. Find the first time the concentration reaches 0.15 kg/L and decide whether this happens before the tank reaches capacity.
4. Find the salt mass when the tank first becomes full. Sketch the concentration over the modeled operation and explain why continuing the same volume formula beyond that time changes the physical problem.

Question 1 – Solution

Strategy. Write separate liquid and salt balances, then use the quotient rule to model concentration in a changing volume.

Step 1: Balance volume and salt. The volume rate is $V' = 3 - 1 = 2$ L/min, so $V = 100 + 2t$ L. Capacity is reached at $t = 50$ min. Up to that time,

$$S' = 0.60 - \frac{S}{100 + 2t}, \quad S(0) = 10.$$

Both terms have units kg/min: salt enters at $(3)(0.20)$ and leaves at $(1)(S/V)$. The pre-overflow model applies for $0 \leq t < 50$, with a continuous value at 50.

Step 2: Derive the concentration equation. Since $S = cV$, $S' = Vc' + cV'$. Hence

$$(100 + 2t)c' = 0.60 - 3c, \quad c(0) = 0.10.$$

Using the integrating factor $(100 + 2t)^{3/2}$ and $c(0) = 0.10$ gives

$$c(t) = 0.20 - 0.10(100/(100 + 2t))^{3/2}.$$

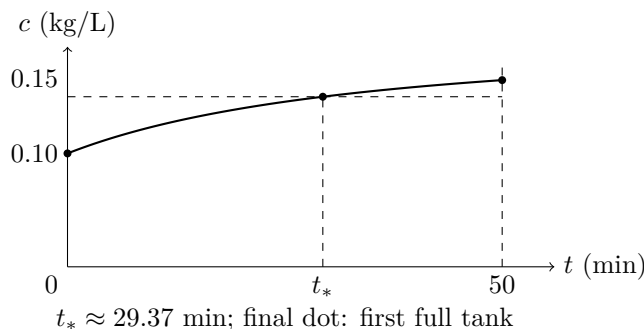
Indeed $c' = 3(0.20 - c)/V$, verifying the concentration balance. Omitting cV' would ignore the dilution caused by increasing volume.

Step 3: Locate the target and capacity values. The concentration increases strictly. Setting $c = 0.15$ gives

$$t_* = 50(2^{2/3} - 1) \approx 29.37 \text{ min} < 50 \text{ min}.$$

At capacity, $c(50) = 0.20 - 0.10/2^{3/2}$, so

$$S(50) = 200c(50) = 40 - 5\sqrt{2} \approx 32.93 \text{ kg}.$$



Step 4: Respect the operating boundary. The final point is the instant the tank becomes full. Beyond it, continued inflow would require overflow or a changed flow rate. Either changes the volume and salt balances, so the formula is not a model of the same operation after 50 min.