

Question 4

Consider the real IVP

$$y' = -\frac{x}{y}, \quad y(0) = 2,$$

whose equation is defined only when $y \neq 0$.

Tasks

1. Obtain an implicit relation and select the explicit branch determined by the initial condition.
2. Find the maximal interval and the limits of both y and y' at its finite endpoints.
3. Decide whether either endpoint can be included, or whether switching to the other square-root branch gives a continuation as a classical solution $y(x)$.
4. Sketch the selected solution and mark its excluded endpoints. Explain why bounded values of y do not guarantee continuation in this example.

Question 4 – Solution

Strategy. Track the forbidden value $y = 0$ and the slope, rather than checking only whether the solution stays bounded.

Step 1: Select the branch. Multiplying by y and integrating gives

$$yy' = -x, \quad x^2 + y^2 = 4.$$

Continuity and $y(0) = 2$ select

$$y = \sqrt{4 - x^2}.$$

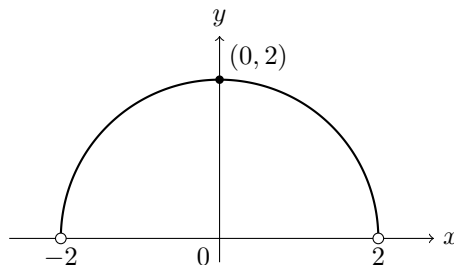
Its positive branch has $y' = -x/\sqrt{4 - x^2}$ and directly satisfies the IVP for $-2 < x < 2$.

Step 2: Test both endpoints. Thus $I = (-2, 2)$, and

	$x \downarrow -2$	$x \uparrow 2$
$y(x)$	0^+	0^+
$y'(x)$	$+\infty$	$-\infty$

At either endpoint the original right-hand side is undefined, and a continuously differentiable extension would also require a finite derivative.

Step 3: Exclude branch switching. The negative square root fails the initial condition. Joining the two semicircles at an endpoint still passes through $y = 0$, and the circle has no real points with $|x| > 2$. Following the circle as a parametrized curve is not an extension as a differentiable graph $y(x)$ on a larger interval.



Step 4: Identify the obstruction. Although $0 < y \leq 2$, the graph approaches the boundary of the equation's domain. Boundedness alone does not keep it inside a region where the differential equation is defined and regular. The open circles indicate excluded endpoints, not additional solution values.