

Question 2

For a real parameter a , consider the equation in the form written:

$$y' + \frac{2x}{x^2 - 9}y = \frac{1}{x^2 - 9}, \quad y(0) = a.$$

The original equation is defined only for $x \neq -3, 3$.

Tasks

1. Use the linear existence theorem to identify an interval on which a unique solution is guaranteed for every a .
2. Find and verify the solution formula in terms of a .
3. Determine which values of a make a singularity of that formula removable, and compute the corresponding finite limit.
4. Determine the maximal interval for the original IVP for every a . Explain whether multiplying the equation by $x^2 - 9$ changes what it means to extend a solution through $x = \pm 3$.

Question 2 – Solution

Strategy. Keep the domain of the given equation separate from the domain of a simplified formula or a multiplied equation.

Step 1: Locate the coefficient interval. Both coefficients are continuous on $(-3, 3)$, the largest coefficient interval containing 0. The linear theorem guarantees a unique solution throughout that interval for every real a .

Step 2: Integrate and verify. On this interval, multiplication and the product rule yield

$$((x^2 - 9)y)' = 1, \quad (x^2 - 9)y = x - 9a.$$

Thus

$$y(x) = \frac{x - 9a}{x^2 - 9}.$$

The value at 0 is a , and differentiating $(x^2 - 9)y = x - 9a$ verifies the differential equation whenever $x \neq \pm 3$.

Step 3: Examine cancellation. At $x = 3$, the numerator vanishes exactly when $a = 1/3$. Then

$$y = \frac{1}{x + 3} \quad (x \neq \pm 3), \quad \lim_{x \rightarrow 3} y = \frac{1}{6}.$$

At $x = -3$, cancellation occurs exactly when $a = -1/3$. Then

$$y = \frac{1}{x - 3} \quad (x \neq \pm 3), \quad \lim_{x \rightarrow -3} y = -\frac{1}{6}.$$

No value of a cancels both factors. For other parameter values the corresponding endpoint is a pole.

Step 4: Use the original domain. For every a , the answer remains $I = (-3, 3)$. Even a finite removable limit does not define the original coefficients at an excluded point.

The multiplied equation $(x^2 - 9)y' + 2xy = 1$ is defined at $x = \pm 3$ and imposes algebraic conditions there. It is equivalent to the original equation only away from those points. A canceled formula can extend through one endpoint as a solution of the multiplied equation, but that is an extension for a different domain of definition.