

Question 5

Consider

$$y' = \frac{x + y - 3}{x - y + 1}, \quad y(2) = 2,$$

on the domain $x - y + 1 \neq 0$.

Tasks

1. Find constants h, k such that $X = x - h, Y = y - k$ remove the constant terms in both affine expressions. Write the shifted equation and initial point.
2. On a neighborhood of that point with $X > 0$, use $v = Y/X$ to obtain an implicit solution in X, Y .
3. Verify the local implicit relation and find the slope at the original initial point.
4. Is $X = 0$ automatically a singular line of the original differential equation? Identify the actual excluded line and explain the difference between these restrictions.

Question 5 – Solution

Strategy. Translate the intersection of the two affine lines to the origin, then apply the homogeneous ratio substitution locally.

Step 1: Choose the translation. The equations $h + k - 3 = 0$ and $h - k + 1 = 0$ give $\boxed{h = 1, k = 2}$. Hence

$$X = x - 1, \quad Y = y - 2, \quad \frac{dY}{dX} = \frac{X + Y}{X - Y}, \quad (X, Y) = (1, 0) \text{ initially.}$$

A translation does not change the derivative because $dX/dx = 1$.

Step 2: Substitute and integrate. With $Y = Xv$,

$$v + Xv' = \frac{1 + v}{1 - v}, \quad \frac{1 - v}{1 + v^2} dv = \frac{dX}{X}.$$

On the local region $X > 0$, integration gives

$$\arctan v - \frac{1}{2} \ln(1 + v^2) = \ln X + C.$$

At $(1, 0)$, $C = 0$. Substituting $v = Y/X$ and simplifying yields

$$\boxed{\arctan(Y/X) - \frac{1}{2} \ln(X^2 + Y^2) = 0}$$

for the local branch through $(1, 0)$ with $X > 0$.

Step 3: Verify and find the slope. For the displayed left side F , differentiation gives

$$F_X = -\frac{X + Y}{X^2 + Y^2}, \quad F_Y = \frac{X - Y}{X^2 + Y^2}.$$

Thus $F_X + F_Y Y' = 0$ recovers the shifted equation wherever $X - Y \neq 0$. At $(1, 0)$, $F_Y = 1$, so a local graph exists and $\boxed{y'(2) = 1}$.

Step 4: Separate coordinate and equation restrictions. The ratio Y/X excludes $X = 0$, namely $x = 1$, but the original equation is defined there whenever $y \neq 2$. Its actual excluded line is $\boxed{x - y + 1 = 0}$, or $y = x + 1$. An alternative local coordinate or angle branch can be needed when the ratio chart fails; that failure alone does not prove that an original solution ends. No global branch claim is made by this local arctangent formula.