

Question 2

On $x > 0$, consider

$$y' = \frac{y}{x} + (y/x - 1)(y/x - 2), \quad y(1) = 3.$$

Tasks

1. Use $v = y/x$ to derive a separable equation. Find every constant value of v before dividing by a factor involving v .
2. Solve the transformed equation, and give the resulting original solution family together with any missing straight-line solution.
3. Select the IVP solution and its maximal interval containing 1.
4. Verify the IVP solution and explain why it cannot cross either straight-line solution within that interval.

Question 2 – Solution

Strategy. Constant ratios correspond to straight lines, so preserve them before separating the nonconstant ratio equation.

Step 1: Transform and preserve the constants. Writing $y = xv$ gives $xv' = (v-1)(v-2)$. The constant ratios $v = 1, 2$ yield the solutions $y = x, y = 2x$. For other ratios,

$$\int \frac{dv}{(v-1)(v-2)} = \int \frac{dx}{x}, \quad \ln \left| \frac{v-2}{v-1} \right| = \ln x + C_0.$$

Thus $(v-2)/(v-1) = Cx$, with nonzero C for nonconstant v , and

$$y = \frac{x(2-Cx)}{1-Cx}.$$

Allowing $C = 0$ restores $y = 2x$; $y = x$ remains a separate solution. Each formula is used only on intervals where its denominator is nonzero.

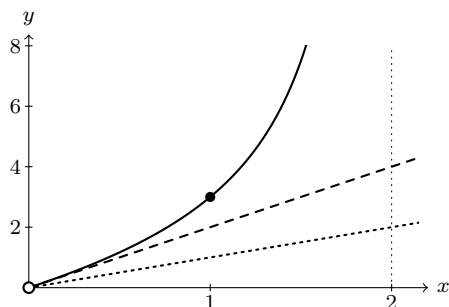
Step 2: Select the IVP branch. The condition $v(1) = 3$ gives $C = 1/2$, hence

$$y = \frac{x(2-x/2)}{1-x/2}, \quad I = (0, 2).$$

The coefficient domain excludes 0, while $y \rightarrow +\infty$ as $x \uparrow 2$.

Step 3: Verify and compare. For $v = (2-Cx)/(1-Cx)$, $v' = C/(1-Cx)^2$, and $(v-1)(v-2) = Cx/(1-Cx)^2 = xv'$. Thus $y' = v + xv'$ recovers the original equation, and the selected initial value is 3.

On $0 < x < 2$, $v = 1 + 1/(1-x/2) > 2$. Therefore $y > 2x > x$ throughout the interval: neither line is crossed.



Solid: IVP solution. Dashed: $y = 2x$. Dotted sloping line: $y = x$. The domain is $x > 0$.