

Question 2

For the IVP

$$y' + y = e^{2x}y^3, \quad y(0) = -1,$$

a student sets $v = y^{-2}$ but writes $v' + 2v = 2e^{2x}$ and then plans to use $y = 1/\sqrt{v}$.

Tasks

1. Find both errors in this procedure and derive the correct equation for v .
2. Solve the linear equation and select the original solution using the initial condition.
3. Find its maximal interval containing 0, explaining why a positive transformed variable does not determine the sign of y .
4. Verify the original IVP, and state whether the opposite sign and the zero function solve the same differential equation.

Question 2 – Solution

Strategy. The derivative of y^{-2} carries a negative factor, and the inverse transformation needs a separately chosen sign.

Step 1: Correct the transformed equation. For $y \neq 0$, divide by y^3 and use $v' = -2y^{-3}y'$. This gives

$$-\frac{1}{2}v' + v = e^{2x}, \quad \boxed{v' - 2v = -2e^{2x}}.$$

The student's two coefficient signs are reversed. Moreover, $v = 1/y^2$ is the same for y and $-y$, so selecting the positive inverse would contradict the negative initial value.

Step 2: Solve and choose the branch. The integrating factor is e^{-2x} :

$$(e^{-2x}v)' = -2, \quad v = e^{2x}(C - 2x).$$

The condition $v(0) = 1$ gives $C = 1$. Continuity and $y(0) = -1$ select

$$\boxed{y = -\frac{e^{-x}}{\sqrt{1-2x}}, \quad I = (-\infty, 1/2)}.$$

Here $v > 0$. At $x \uparrow 1/2$, the solution tends to $-\infty$, so there is no finite extension; for $x > 1/2$, this v cannot equal $1/y^2$ for real y .

Step 3: Verify and distinguish other solutions. On I , logarithmic differentiation of $|y|$ gives

$$\frac{y'}{y} = -1 + \frac{1}{1-2x}.$$

Consequently $y' + y = y/(1-2x) = e^{2x}y^3$, and the initial value is -1 . The opposite branch $+e^{-x}/\sqrt{1-2x}$ also solves the differential equation, with initial value 1 . The zero function solves it as well but is omitted by y^{-2} . Neither of those alternatives solves the stated IVP.