

Question 10

For the exact form

$$\omega = (2xy + e^x) dx + (x^2 + 2y) dy,$$

compare two routes from $A = (0, 0)$ to $B = (1, 1)$. Route 1 goes horizontally to $(1, 0)$ and then vertically to B ; Route 2 goes vertically to $(0, 1)$ and then horizontally to B .

For an axis-aligned segment, define its accumulated value as the integral of the coefficient of dx along a horizontal segment, or of the coefficient of dy along a vertical segment, with the stated orientation.

Tasks

1. Construct a potential F normalized by $F(0, 0) = 0$.
2. Compute both route totals directly as sums of ordinary single-variable integrals.
3. Explain their equality using the chain rule and find the total around the closed route that follows Route 1 forward and Route 2 backward.
4. For the differential equation $\omega = 0$, decide whether one solution curve could pass through both A and B . Distinguish this question from the equality of the two route totals.

Question 10 – Solution

Strategy. A potential measures accumulated change along a route; a solution of the equation $\omega = 0$ must instead keep that potential constant.

Step 1: Construct a normalized potential. The cross partials are $2x$. Integrating M gives $F = x^2y + e^x + h(y)$; matching N requires $h' = 2y$. Normalization yields

$$\boxed{F = x^2y + e^x + y^2 - 1}, \quad F(B) = e + 1.$$

Step 2: Compute the routes directly. On Route 1, the horizontal and vertical contributions are

$$\int_0^1 e^x dx = e - 1, \quad \int_0^1 (1 + 2y) dy = 2,$$

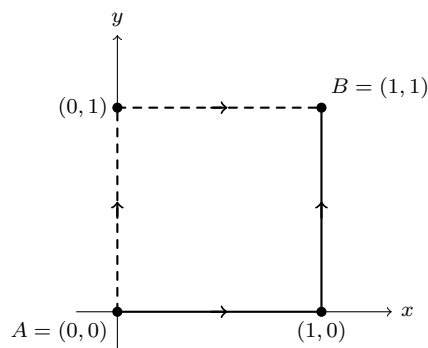
so its total is $e + 1$. On Route 2 they are

$$\int_0^1 2y dy = 1, \quad \int_0^1 (2x + e^x) dx = e,$$

again totaling $\boxed{e + 1}$.

Step 3: Explain equality and the closed route. Along a differentiable parametrized segment, $dF/dt = F_x x'(t) + F_y y'(t) = Mx' + Ny'$. Integration gives the endpoint difference of F ; these differences telescope across joined segments. Both routes therefore give $F(B) - F(A) = e + 1$. Reversing a route negates its integrals, so the closed-route total is $\boxed{0}$.

Step 4: Distinguish routes from solutions. A solution curve of $\omega = 0$ has $dF/dt = 0$ and stays on one level. Since $F(A) = 0$ and $F(B) = e + 1 \neq 0$, no such curve can pass through both points. Equal route totals express endpoint dependence; they do not make either route a solution of the differential equation.



Solid: Route 1. Dashed: Route 2. Arrows indicate travel from A toward B .