

Question 3

Consider an equation whose right-hand side is undefined on the line $y = 2$:

$$y' = -\frac{e^x}{y-2}, \quad y(0) = 1.$$

Tasks

1. Separate the equation and obtain an implicit relation.
2. Select the correct explicit branch and determine its maximal interval containing 0.
3. Find the limiting value of y and the behavior of y' at the finite endpoint. Decide whether that endpoint can be included.
4. Another student takes the opposite square-root sign, then proposes switching signs at the endpoint. Test both claims against the equation and initial condition.

Question 3 – Solution

Strategy. Integration gives a squared relation; the initial condition and the excluded line determine which part is a solution.

Step 1: Integrate and choose the sign.

$$(y - 2) dy = -e^x dx, \quad \frac{1}{2}(y - 2)^2 = -e^x + C.$$

At $(0, 1)$, $C = 3/2$, so $(y - 2)^2 = 3 - 2e^x$. Since $y(0) - 2 = -1$, continuity selects

$$y = 2 - \sqrt{3 - 2e^x}, \quad I = (-\infty, \ln(3/2)).$$

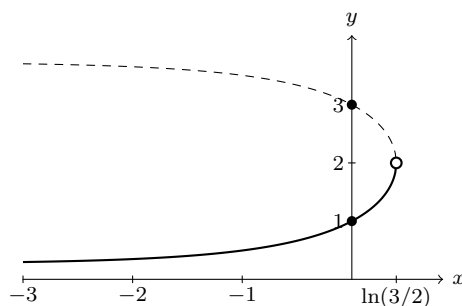
The radicand must be strictly positive: zero would give the forbidden value $y = 2$.

Step 2: Check behavior and verify. Throughout I ,

$$y' = \frac{e^x}{\sqrt{3 - 2e^x}} = -\frac{e^x}{y - 2} > 0.$$

As $x \rightarrow -\infty$, $y \rightarrow 2 - \sqrt{3}$. As $x \uparrow \ln(3/2)$, $y \rightarrow 2$ but $y' \rightarrow +\infty$. A finite limit of y alone does not permit extension: the equation is undefined at the endpoint and a finite derivative is impossible there.

Step 3: Audit the other branch. The branch $2 + \sqrt{3 - 2e^x}$ satisfies the differential equation on I , but has $y(0) = 3$, not 1. Switching signs would still pass through $y = 2$, and for larger x the radicand is negative. Neither proposal extends the given real solution.



Solid: $y(0) = 1$. Dashed: $y(0) = 3$. The common limiting point is excluded.