

Question 10

A scaled temperature model uses time x and temperature y :

$$y' = T(x) - y, \quad T(x) = 2 + \sin x.$$

The prescribed ambient temperature T varies in time. A periodic response is proposed:

$$r(x) = 2 + \frac{\sin x - \cos x}{2}.$$

Tasks

1. Interpret the slope signs above and below $y = T(x)$. Compute the slopes at $(0, 1)$, $(0, 3)$, and $(\pi/2, 2)$.
2. Test whether the ambient-temperature curve itself solves the equation. Verify that r does solve it.
3. On $0 \leq x \leq 2\pi$, find the maximum and minimum of r , including their times. Compare the time of its maximum with the time of the ambient maximum.
4. Draw the field, the ambient curve, and the response in your solution. Explain why a response extremum occurs where the response meets the zero-slope curve, although that curve is not itself a solution.

Question 10 – Solution

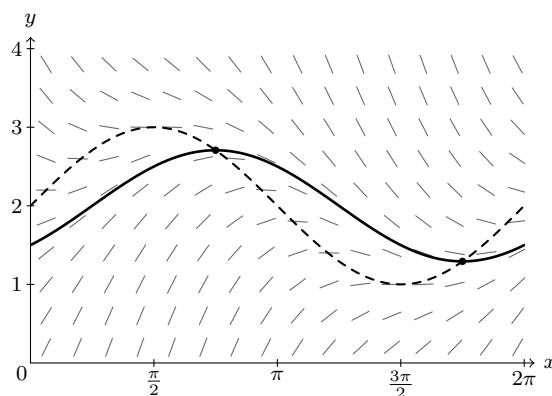
Strategy. Interpret the moving zero-slope curve as a temperature comparison, then verify the response and locate its horizontal tangents.

Step 1: Heating and cooling. Below $y = T(x)$, $y' > 0$ and the object warms; above it, $y' < 0$ and it cools. At the three listed points, the slopes are $\boxed{1, -1, 1}$.

Step 2: Check both curves. Along $y = T(x)$ the right-hand side is zero, but $T'(x) = \cos x$ is not identically zero on any interval. Thus T is not a solution. By contrast,

$$r' = \frac{\cos x + \sin x}{2} = 2 + \sin x - r,$$

so r is a solution for all real x .



Solid: response r . Dashed: ambient temperature T . Dots: response extrema.

Step 3: Extrema and delay. Rewrite

$$r(x) = 2 + \frac{1}{\sqrt{2}} \sin(x - \pi/4).$$

On the stated interval, its extreme values are

$$r_{\max} = 2 + \frac{1}{\sqrt{2}} \text{ at } x = \frac{3\pi}{4}, \quad r_{\min} = 2 - \frac{1}{\sqrt{2}} \text{ at } x = \frac{7\pi}{4}.$$

The endpoint values both equal $3/2$ and lie between these extremes. The ambient maximum occurs at $x = \pi/2$, so the response maximum follows it by $\boxed{\pi/4}$ time units.

Step 4: Field interpretation. At a response extremum, $r' = T - r = 0$, so the response meets the zero-slope curve. Elsewhere it need not equal T . A locus of zero assigned slopes can move with time without being a solution curve itself.