

Question 6

Compare two equations on the same coordinate scale:

$$(A) \quad y' = y, \quad (B) \quad y' = 3y.$$

Both are assigned the initial value $y(0) = 1$. Candidate solutions are supplied:

$$u(x) = e^x, \quad v(x) = e^{3x}.$$

Tasks

1. Determine the zero-slope locations and slope signs for both fields. Are these facts enough to make the two fields identical?
2. At height $y = 1$, compute each slope and its angle with the positive x -axis. If the two fields use equal-length segments, explain how steepness should be represented.
3. Verify the two candidates and calculate when each first reaches $y = 2$ for $x > 0$.
4. Draw the two fields and their respective verified curves on identical axes in your solution. State precisely which geometric information is lost by recording only slope signs.

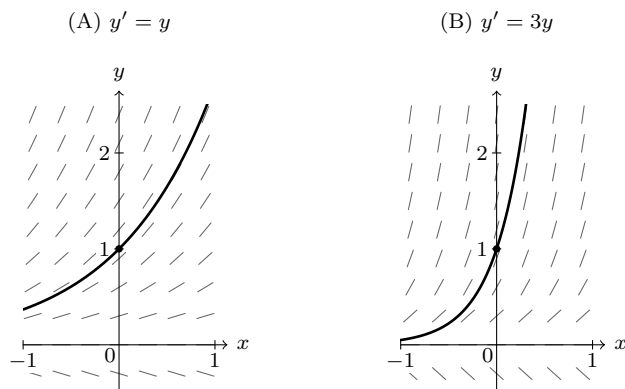
Question 6 – Solution

Strategy. Separate slope sign from slope magnitude. Equal-length field segments encode magnitude through their angle, not their length.

Step 1: Shared and different information. Both fields have horizontal segments on $y = 0$, positive slopes above it, and negative slopes below it. But at $y = 1$ their slopes are 1 and 3, so the fields are different. The corresponding angles are

$$\theta_A = \arctan(1) = 45^\circ, \quad \theta_B = \arctan(3) \approx 71.6^\circ.$$

A segment of total length ℓ and slope m can use the displacement $\ell(1, m)/\sqrt{1 + m^2}$. This preserves its slope while controlling its length.



Solid curves: e^x on the left and e^{3x} on the right; identical axis scales.

Step 2: Verify and compare growth. Directly,

$$u' = e^x = u, \quad v' = 3e^{3x} = 3v, \quad u(0) = v(0) = 1.$$

Both increase strictly. Solving $e^x = 2$ and $e^{3x} = 2$ gives their first hitting times

$$x_A = \ln 2, \quad x_B = \frac{\ln 2}{3}.$$

Thus (B) reaches the same height in one-third the time.

Step 3: What signs omit. Signs indicate rising, falling, or horizontal tangents. They do not specify tangent angles, rates of change, or travel times between heights. Identical sign regions and the same equilibrium line do not determine the same solution curves.