

Question 5

Consider the equation

$$y' = \frac{x}{y}, \quad y \neq 0,$$

and a solution through $(0, 1)$.

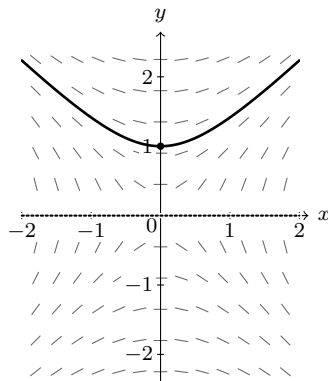
Tasks

1. Determine the slope sign in each open quadrant. Find every point in the domain with a horizontal field segment.
2. Explain why no segment should be placed at $(1, 0)$, even though nearby segments become steep.
3. Along a solution, differentiate $y^2 - x^2$. Use the initial point to identify an explicit solution branch and verify it directly. State its largest open interval.
4. Draw the field and the selected branch in your solution, marking the excluded line $y = 0$. Explain why that excluded line does not prevent this particular solution from existing for every real x .

Question 5 – Solution

Strategy. Respect the equation's domain before drawing. Use a constant quantity along a solution to identify the curve through the given point.

Step 1: Signs and missing points. Slopes are positive in Quadrants I and III, and negative in II and IV. Horizontal segments occur exactly where $x = 0$ and $y \neq 0$. The expression x/y is undefined on all of $y = 0$; a vertical mark at $(1, 0)$ would falsely assign a direction to a point outside the domain.



Solid: $y = \sqrt{1 + x^2}$. Dotted horizontal line: excluded $y = 0$.

Step 2: Identify and verify the branch. Wherever a solution is defined,

$$\frac{d}{dx}(y^2 - x^2) = 2yy' - 2x = 2y\frac{x}{y} - 2x = 0.$$

Thus $y^2 - x^2$ is constant on its interval. At $(0, 1)$ it equals 1. Continuity and the positive initial value select

$$y = \sqrt{1 + x^2}, \quad -\infty < x < \infty.$$

Indeed $y' = x/\sqrt{1 + x^2} = x/y$ and $y(0) = 1$.

Step 3: Interval and geometry. This branch is smooth and satisfies $y \geq 1$ for every real x , so it never reaches the excluded axis. The entire real line is therefore its largest open interval. It decreases for $x < 0$ and increases for $x > 0$, with a minimum at $(0, 1)$, matching the field.