

Question 1

Consider the direction field for

$$y' = x - y.$$

An *isocline of slope k* is the set of points where the equation assigns slope k . A direction field consists of short line segments with the prescribed slopes; the segments are not themselves whole solution curves.

Tasks

1. Calculate the slopes at $(-1, 1)$, $(0, 1)$, $(1, 1)$, and $(1, -1)$, and describe the orientation of each segment.
2. Find the isoclines of slopes -1 , 0 , and 1 . Determine the regions of positive and negative slope.
3. Draw a direction field on $-2 \leq x \leq 2$, $-2 \leq y \leq 2$ in your solution. Mark the three isoclines and distinguish them from solution curves.
4. A solution passes through $(0, 1)$. Find its tangent line there and decide whether it initially rises or falls as x increases. Does the tangent line itself solve the equation on an interval?

Question 1 – Solution

Strategy. Evaluate $x - y$ at points, then hold its value fixed to find isoclines. Test a proposed curve by comparing its derivative with the assigned slope.

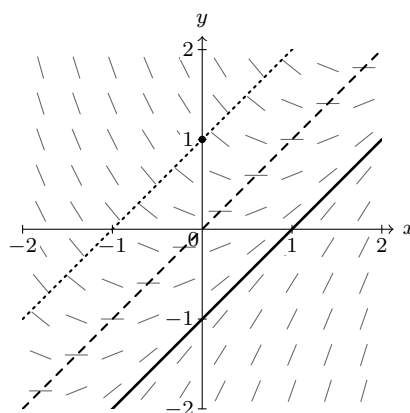
Step 1: Slopes and regions. The four slopes, in the listed order, are

$$\boxed{-2, \quad -1, \quad 0, \quad 2.}$$

Thus the first two segments fall to the right, the third is horizontal, and the fourth rises. The isocline equation is $x - y = k$, or $y = x - k$. Hence

$$k = -1 : y = x + 1, \quad k = 0 : y = x, \quad k = 1 : y = x - 1.$$

Below $y = x$ the slopes are positive; above it they are negative.



Solid: $y = x - 1$; dashed: $y = x$; dotted: $y = x + 1$. Dot: $(0, 1)$.

Step 2: Isoclines are not automatically solutions. Each displayed isocline has geometric slope 1. On $y = x + 1$ and $y = x$, the field instead prescribes -1 and 0 , so these lines are not solutions. On $y = x - 1$, the prescribed slope is 1 , so that particular isocline also happens to be a solution.

Step 3: Tangent versus solution. At $(0, 1)$ the slope is -1 , giving $\boxed{y = 1 - x}$ as the tangent line. The solution initially falls. Along this line, however, $x - y = 2x - 1$, which equals the line's derivative -1 only at $x = 0$. Thus the tangent line does not solve the equation on any open interval.