

Question 3

An implicit relation and an initial value are given by

$$x^2 + y^2 = 9, \quad y(0) = 3.$$

The proposed differential equation is

$$y' = -\frac{x}{y}.$$

Tasks

1. Identify the explicit branch selected by the initial value and verify the differential equation.
2. Find the largest open interval containing 0 on which that branch is a classical solution of the displayed equation.
3. Explain why the entire circle is not one solution function $y(x)$, and why adding the endpoints of the upper semicircle does not extend the solution interval.
4. If the initial value is changed to $y(0) = -3$, identify the corresponding branch and its interval. Sketch both branches in your solution.

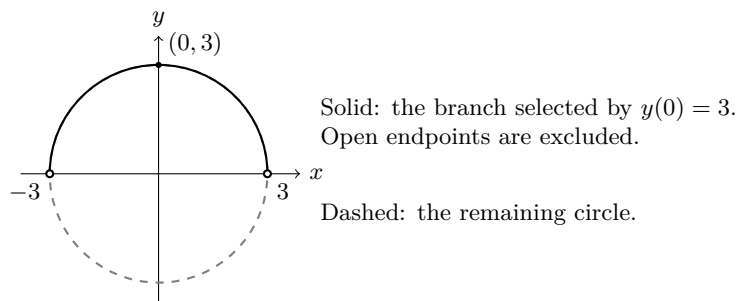
Question 3 – Solution

Strategy. An implicit curve can contain multiple function branches. Select the branch using the initial value, then check both differentiability and the equation's denominator.

Step 1: Select and verify. Solving for y gives $y = \pm\sqrt{9-x^2}$. Since $y(0) = 3$, choose

$$y(x) = \sqrt{9-x^2}, \quad -3 < x < 3.$$

On this interval, $y' = -x/\sqrt{9-x^2} = -x/y$, and $y(0) = 3$.



Step 2: Interval and endpoints. At $x = \pm 3$, the branch has $y = 0$, so $-x/y$ is undefined and the derivative is not finite. For $|x| > 3$, this branch is not real. Thus the largest open interval containing 0 is exactly $(-3, 3)$.

Step 3: Relation versus function. For each $-3 < x < 3$, the circle has two distinct y -values. The whole circle fails the single-valued requirement for a function $y(x)$. Even the undivided equation $yy' = -x$ cannot hold with a finite derivative at $(\pm 3, 0)$: it would require $0 = \mp 3$.

Step 4: Other initial value. For $y(0) = -3$,

$$y(x) = -\sqrt{9-x^2}, \quad -3 < x < 3.$$

Its derivative is $x/\sqrt{9-x^2} = -x/y$, verifying the lower branch separately.