

Question 1

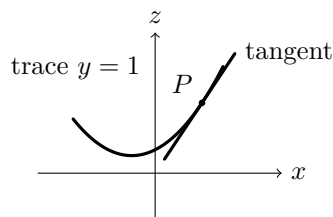
For $f(x, y) = x^2 + 3xy - y^2$ at $P = (1, 2, 3)$:

Tasks

1. Find f_x, f_y at $(1, 2)$.
2. Interpret them as trace slopes.
3. Write both trace tangent lines.

Question 1 – Solution

Strategy. Hold one input fixed at a time.



Step 1: Rates $f_x = 2x + 3y$, $f_y = 3x - 2y$, hence $\boxed{f_x(1, 2) = 8}$ and $\boxed{f_y(1, 2) = -1}$.

Step 2: Meaning With $y = 2$, height rises 8 units per x unit; with $x = 1$, it falls 1 unit per y unit.

Step 3: Lines $\boxed{y = 2, z - 3 = 8(x - 1)}$ and $\boxed{x = 1, z - 3 = -(y - 2)}$.

Question 2

Temperature is $T(x, y) = 80 - 2x^2 - y^2$ degrees Celsius, with distance in centimeters.

Tasks

1. Find $T_x(3, 4), T_y(3, 4)$ with units.
2. Interpret signs and magnitudes.
3. Estimate separate changes for 0.05 cm moves in $+x$ and $+y$.

Question 2 – Solution

Strategy. Read each partial as temperature change per centimeter with the other coordinate fixed.

Step 1: Rates $T_x = -4x$, $T_y = -2y$, so $T_x = -12\text{ }^\circ\text{C/cm}$ and $T_y = -8\text{ }^\circ\text{C/cm}$ at $(3, 4)$.

Step 2: Meaning Both directions cool; the x trace is locally steeper.

Step 3: Estimates $\Delta T_x \approx -12(.05) = -0.60^\circ\text{C}$ and $\Delta T_y \approx -8(.05) = -0.40^\circ\text{C}$.

Question 3

Revenue is $R(p, a) = 1200p - 30p^2 + 50a - 2a^2 + 4pa$, where p is dollars and a is hundreds of advertising dollars.

Tasks

1. Compute $R_p(10, 5), R_a(10, 5)$.
2. State units.
3. Interpret both values.

Question 3 – Solution

Strategy. Attach output units per input unit.

Step 1: Compute $R_p = 1200 - 60p + 4a$ and $R_a = 50 - 4a + 4p$, so $\boxed{R_p(10, 5) = 620}$ and $\boxed{R_a(10, 5) = 70}$.

Step 2: Units These are revenue dollars per price dollar and revenue dollars per hundred advertising dollars.

Step 3: Meaning Locally, a \$1 price rise predicts \$620 more revenue; an added \$100 advertising predicts \$70 more, with the other input fixed.

Question 4

Pressure is $P(V, T) = 8T/V$ kPa, with V in liters and T in kelvins.

Tasks

1. Find $P_V(4, 300)$, $P_T(4, 300)$.
2. Supply units.
3. Interpret each sign.

Question 4 – Solution

Strategy. Vary one physical input while fixing the other.

Step 1: Volume $P_V = -8T/V^2$, so $P_V(4, 300) = -150 \text{ kPa/L}$.

Step 2: Temperature $P_T = 8/V$, so $P_T(4, 300) = 2 \text{ kPa/K}$.

Step 3: Meaning Added volume lowers pressure locally; added temperature raises it. The statements are instantaneous and hold the other variable fixed.

Question 5

At a terrain point, $f_x = -0.25$ and $f_y = 0.60$, with all distances in meters.

Tasks

1. Interpret both as rise/run.
2. Find signed trace angles.
3. Identify the steeper trace.

Question 5 – Solution

Strategy. A trace slope is the tangent of its signed inclination.

Step 1: Rise/run A one-meter positive x move predicts -0.25 m elevation change; a positive y move predicts 0.60 m, separately.

Step 2: Angles $\alpha_x = \arctan(-.25) \approx -14.0^\circ$ and $\alpha_y = \arctan(.60) \approx 31.0^\circ$.

Step 3: Compare Since $.60 > .25$, the y trace is steeper; its positive sign indicates ascent.

Question 6

Near $(2, 3)$, the columns give x and the rows give y :

	1.9	2.0	2.1
2.9	7.62	7.80	7.98
3.0	7.71	7.90	8.09
3.1	7.80	8.00	8.20

Tasks

1. Estimate $f_x(2, 3)$ centrally.
2. Estimate $f_y(2, 3)$ centrally.
3. Interpret both.

Question 6 – Solution

Strategy. Use the center row and column.

Step 1: x rate $f_x \approx (8.09 - 7.71)/.2 = \boxed{1.90}$.

Step 2: y rate $f_y \approx (8.00 - 7.80)/.2 = \boxed{1.00}$.

Step 3: Meaning Output rises about 1.90 per x unit with y fixed and 1.00 per y unit with x fixed. Symmetric differences reduce one-sided bias.

Question 7

Adjacent contours differ by 6, with labels increasing in both positive coordinate directions. Along $+x$ they are 2 units apart; along $+y$ they are 3 units apart.

Tasks

1. Estimate both partial derivatives.
2. Relate spacing to magnitude.
3. Explain how decreasing labels change signs.

Question 7 – Solution

Strategy. Divide contour-value change by coordinate distance.

Step 1: Estimates $f_x \approx 6/2 = 3$ and $f_y \approx 6/3 = 2$.

Step 2: Spacing Equal output change over less distance gives larger slope, so the x trace is steeper.

Step 3: Sign If labels decrease in a positive direction, the corresponding partial is negative. Spacing controls magnitude; label order controls sign.

Question 8

Cylinder volume is $V(r, h) = \pi r^2 h$.

Tasks

1. Interpret V_r, V_h geometrically.
2. Evaluate at $(3, 10)$ cm.
3. Estimate separate effects of 0.1 cm increases.

Question 8 – Solution

Strategy. Connect derivatives to familiar areas.

Step 1: Geometry $V_r = 2\pi rh$ is circumference times height; $V_h = \pi r^2$ is base area.

Step 2: Values $V_r = 60\pi \text{ cm}^2$, $V_h = 9\pi \text{ cm}^2$.

Step 3: Estimates $\Delta V_r \approx 6\pi \text{ cm}^3$ and $\Delta V_h \approx 0.9\pi \text{ cm}^3$. The radial change has the larger first-order effect.

Question 9

At one point a concentration field has $C_x = 4$, $C_y = 0$, $C_t = -1.5$ in appropriate mg/L per meter or minute units.

Tasks

1. Interpret each.
2. Explain what $C_y = 0$ does not imply.
3. Predict separate changes for .2 m in $+x$, .2 m in $+y$, and 2 minutes.

Question 9 – Solution

Strategy. Preserve held-fixed conditions and units.

Step 1: Meaning Concentration rises 4 mg/L per x meter, has zero instantaneous y slope, and falls 1.5 mg/L per minute.

Step 2: Caution $C_y = 0$ at one point does not mean independence from y nearby.

Step 3: Predictions $\Delta C_x \approx .8 \text{ mg/L}$, $\Delta C_y \approx 0$, and $\Delta C_t \approx -3.0 \text{ mg/L}$.

Question 10

At the same input, models satisfy $A_x = 5$, $A_y = -2$ and $B_x = -1$, $B_y = -2$.

Tasks

1. Compare positive- x responses.
2. Compare positive- y responses.
3. State what cannot be concluded from these data.

Question 10 – Solution

Strategy. Partial derivatives give local coordinate-specific rates.

Step 1: x A locally rises at 5 per unit; B falls at 1 per unit. Their signs oppose and A has greater absolute sensitivity.

Step 2: y Both locally fall at 2 per positive y unit.

Step 3: Limits We cannot infer $A > B$, global monotonicity, or exact finite changes without more information.

The data justify local rate comparisons only.