

Question 8

For a constant a , let

$$\mathbf{F}_a(x, y, z) = \langle ax, 2y, 3z \rangle.$$

The measured outward flux through the unit sphere is 24π . Determine a .

Tasks

1. Express the divergence in terms of a .
2. Use the flux measurement and the unit-ball volume.
3. Verify the recovered value directly on the sphere.

Question 8 – Solution

Strategy. The divergence is constant, so the measured flux gives a linear equation for the unknown coefficient.

Step 1: Apply the theorem

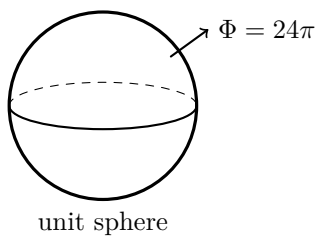
$$\nabla \cdot \mathbf{F}_a = a + 2 + 3 = a + 5.$$

The unit ball has volume $4\pi/3$, so

$$24\pi = (a + 5) \frac{4\pi}{3}.$$

Therefore $a + 5 = 18$, and

$$\boxed{a = 13}.$$



Step 2: Direct verification On the unit sphere, $\mathbf{n} = \langle x, y, z \rangle$, so with $a = 13$,

$$\mathbf{F}_{13} \cdot \mathbf{n} = 13x^2 + 2y^2 + 3z^2.$$

By spherical symmetry,

$$\iint_S x^2 dS = \iint_S y^2 dS = \iint_S z^2 dS = \frac{4\pi}{3}.$$

Hence the direct flux is

$$(13 + 2 + 3) \frac{4\pi}{3} = 24\pi.$$

Verification The recovered parameter reproduces the stated measurement by both methods.