

Question 5

Let S be the curved upper hemisphere $x^2 + y^2 + z^2 = 4$, $z \geq 0$, oriented outward. For

$$\mathbf{F}(x, y, z) = \langle x, y, z + 1 \rangle,$$

find the flux through S by closing it with its base disk.

Tasks

1. Apply the Divergence Theorem to the closed upper half-ball.
2. Compute the flux through the base disk with its outward normal.
3. Isolate and verify the curved-surface flux.

Question 5 – Solution

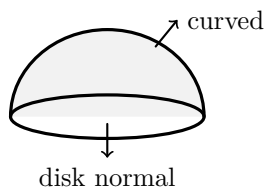
Strategy. Add the base disk, compute the total closed flux, and subtract the disk contribution with careful attention to its downward normal.

Step 1: Closed flux The divergence is

$$\nabla \cdot \mathbf{F} = 3.$$

The upper half-ball has volume $\frac{1}{2} \cdot \frac{4}{3}\pi(2^3) = 16\pi/3$. Therefore

$$\Phi_{\text{closed}} = 3 \left(\frac{16\pi}{3} \right) = 16\pi.$$



Step 2: Base disk On $z = 0$, the outward normal for the half-ball is $-\mathbf{k}$. Since $\mathbf{F} = \langle x, y, 1 \rangle$ there,

$$\Phi_{\text{disk}} = \iint_D (-1) dA = -4\pi.$$

Step 3: Curved part Because $\Phi_{\text{closed}} = \Phi_S + \Phi_{\text{disk}}$,

$$\boxed{\Phi_S = 16\pi - (-4\pi) = 20\pi}.$$

Verification On the sphere, $\mathbf{F} = \langle x, y, z \rangle + \mathbf{k}$. The radial part contributes $2(8\pi) = 16\pi$ over the hemisphere, and the constant vertical part contributes its projected area 4π , totaling 20π .