

Question 4

Let E be the spherical shell $a \leq \sqrt{x^2 + y^2 + z^2} \leq b$, where $0 < a < b$, with outward orientation on its entire boundary. For

$$\mathbf{F} = \frac{\langle x, y, z \rangle}{(x^2 + y^2 + z^2)^{3/2}},$$

find the net flux through the shell boundary.

Tasks

1. Compute the divergence in the shell.
2. Track the outward normal on both spherical boundary components.
3. Reconcile the separate nonzero fluxes with the net result.

Question 4 – Solution

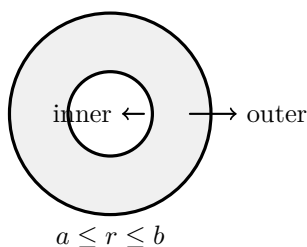
Strategy. The origin is excluded from the shell, so the field is smooth there and has zero divergence; the inner normal points toward the origin.

Step 1: Apply the theorem For $r > 0$,

$$\nabla \cdot \left(\frac{\langle x, y, z \rangle}{r^3} \right) = 0.$$

Therefore

$$\boxed{\iint_{\partial E} \mathbf{F} \cdot \mathbf{n} \, dS = \iiint_E 0 \, dV = 0.}$$



Step 2: Outer sphere At $r = b$, $\mathbf{F}$ points in the outward-normal direction with normal component $1/b^2$. Hence

$$\Phi_{r=b} = \frac{1}{b^2}(4\pi b^2) = 4\pi.$$

Step 3: Inner sphere The outward normal for the shell at $r = a$ points into the cavity, namely $-\mathbf{e}_r$. Thus

$$\Phi_{r=a} = -\frac{1}{a^2}(4\pi a^2) = -4\pi.$$

The two boundary components sum to zero.

Verification The cancellation depends on reversing the normal at the inner boundary; using the radial normal on both spheres would not describe the shell's outward orientation.