

Question 3

Let E be the solid cylinder $x^2 + y^2 \leq 4$, $0 \leq z \leq 3$, and let $S = \partial E$ be oriented outward. For

$$\mathbf{F}(x, y, z) = \langle xz, yz, z^2 \rangle,$$

find the net outward flux and verify it by splitting S into its side and caps.

Tasks

1. Apply the Divergence Theorem in cylindrical coordinates.
2. Compute the flux through the curved side and both caps directly.
3. Compare the totals.

Question 3 – Solution

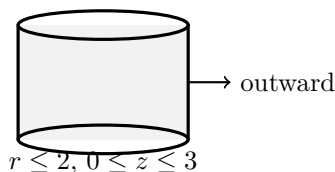
Strategy. First obtain the net flux from the divergence, then use the simple normals of a cylinder as an independent check.

Step 1: Volume integral Since

$$\nabla \cdot \mathbf{F} = z + z + 2z = 4z,$$

we obtain

$$\begin{aligned} \iint_S \mathbf{F} \cdot \mathbf{n} \, dS &= \int_0^{2\pi} \int_0^2 \int_0^3 4z \, r \, dz \, dr \, d\theta \\ &= \boxed{72\pi}. \end{aligned}$$



Step 2: Curved side On $r = 2$, the outward vector element is $\langle 2 \cos \theta, 2 \sin \theta, 0 \rangle \, d\theta \, dz$. Thus the dot product is $4z$, and

$$\Phi_{\text{side}} = \int_0^{2\pi} \int_0^3 4z \, dz \, d\theta = 36\pi.$$

Step 3: Caps On the top $z = 3$, $\mathbf{F} \cdot \mathbf{k} = 9$, so $\Phi_{\text{top}} = 9(4\pi) = 36\pi$. On the bottom, $z = 0$, so $\Phi_{\text{bottom}} = 0$.

$$\boxed{\Phi_{\text{side}} + \Phi_{\text{top}} + \Phi_{\text{bottom}} = 72\pi}.$$

Verification The direct total matches the volume-integral result exactly.