

Question 2

Let $E = [0, 1] \times [0, 2] \times [0, 3]$ and let $S = \partial E$ have outward orientation. For

$$\mathbf{F}(x, y, z) = \langle x^2, yz, z \rangle,$$

compute the total outward flux through the six faces.

Tasks

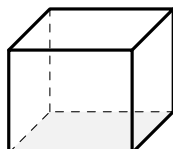
1. Find the divergence.
2. Evaluate the resulting triple integral by separating its terms.
3. Explain why this is preferable to six direct surface integrals.

Question 2 – Solution

Strategy. The box is closed and the field is smooth, so one volume integral replaces six oriented face calculations.

Step 1: Divergence

$$\nabla \cdot \mathbf{F} = 2x + z + 1.$$



$1 \times 2 \times 3$ box

Step 2: Integrate term by term The three contributions over E are

$$\begin{aligned}\iiint_E 2x \, dV &= \left(\int_0^1 2x \, dx \right) (2)(3) = 6, \\ \iiint_E z \, dV &= (1)(2) \int_0^3 z \, dz = 9, \\ \iiint_E 1 \, dV &= (1)(2)(3) = 6.\end{aligned}$$

Hence

$$\iint_S \mathbf{F} \cdot \mathbf{n} \, dS = \iiint_E (2x + z + 1) \, dV = 21.$$

Step 3: Interpret A direct method would require a different outward normal and parameter domain on every face. The divergence automatically combines all six signed contributions.

Verification The divergence is positive throughout E , so a positive net outward flux is consistent.