

Question 1

Let S be the sphere $x^2 + y^2 + z^2 = 4$, oriented outward, and let

$$\mathbf{F}(x, y, z) = \langle x, y, z \rangle.$$

Tasks

1. Compute $\nabla \cdot \mathbf{F}$.
2. Use the Divergence Theorem to find the outward flux through S .
3. Verify the result directly from the geometry of the sphere.

Question 1 – Solution

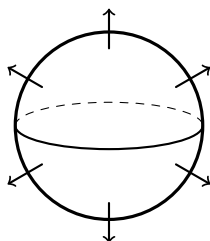
Strategy. Replace the closed-surface flux by a volume integral over the radius-2 ball.

Step 1: Divergence

$$\boxed{\nabla \cdot \mathbf{F} = 1 + 1 + 1 = 3}.$$

If E is the ball bounded by S , then

$$\text{vol}(E) = \frac{4}{3}\pi(2^3) = \frac{32\pi}{3}.$$



outward flux through $r = 2$

Step 2: Apply the theorem

$$\boxed{\iint_S \mathbf{F} \cdot \mathbf{n} \, dS = \iiint_E 3 \, dV = 3 \left(\frac{32\pi}{3} \right) = 32\pi}.$$

Step 3: Direct check On the sphere, the outward unit normal is $\mathbf{n} = \langle x, y, z \rangle / 2$. Therefore

$$\mathbf{F} \cdot \mathbf{n} = \frac{x^2 + y^2 + z^2}{2} = 2.$$

Multiplying by the surface area $4\pi(2^2) = 16\pi$ gives $2(16\pi) = 32\pi$.

Verification The radial field points outward everywhere, so the positive sign is required.