

Question 8

Let C be the boundary of the part of the plane $z = 2x - y$ above

$$0 \leq x \leq 1, \quad 0 \leq y \leq 2,$$

oriented consistently with the normal having positive z -component. Let

$$\mathbf{F} = \frac{1}{2} \langle 2z - 3y, 3x - z, y - 2x \rangle.$$

Compute $\oint_C \mathbf{F} \cdot d\mathbf{r}$.

Tasks

1. Compute the curl of $\mathbf{F}$.
2. Find the oriented vector surface element for the plane.
3. Apply Stokes' Theorem and verify the orientation.

Question 8 – Solution

Strategy. The field was chosen to have constant curl, while the plane has a constant vector area element.

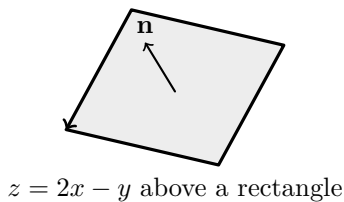
Step 1: Curl Direct differentiation gives

$$\nabla \times \mathbf{F} = \langle 1, 2, 3 \rangle.$$

Step 2: Surface element With $z = g(x, y) = 2x - y$,

$$\mathbf{n} dS = \langle -g_x, -g_y, 1 \rangle dA = \langle -2, 1, 1 \rangle dA.$$

Its third component is positive, as required.



Step 3: Apply Stokes The curl flux density is constant:

$$\langle 1, 2, 3 \rangle \cdot \langle -2, 1, 1 \rangle = 3.$$

The projected rectangle has area 2, so

$$\oint_C \mathbf{F} \cdot d\mathbf{r} = \iint_R 3 dA = 6.$$

Verification Reversing the traversal of all four edges would correspond to the negative normal and would change the value to -6 .