

Question 5

Let S be the annulus $1 \leq x^2 + y^2 \leq 4$ in the plane $z = 0$, oriented upward. Its boundary consists of an outer circle C_o and inner circle C_i . For

$$\mathbf{F} = \left\langle -\frac{y}{2}, \frac{x}{2}, 0 \right\rangle,$$

evaluate the circulation around the fully oriented boundary ∂S .

Tasks

1. Determine the induced direction on each boundary component.
2. Apply Stokes' Theorem to the annulus.
3. Verify the result from the two separate circle integrals.

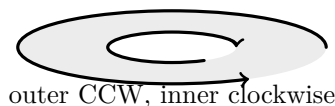
Question 5 – Solution

Strategy. For an upward annulus, the outer boundary is counterclockwise and the inner boundary is clockwise.

Step 1: Orient the boundary The right-hand rule gives

$$\partial S = C_o - C_i^{\text{CCW}}.$$

Equivalently, traverse the outer circle counterclockwise and the inner circle clockwise.



Step 2: Apply Stokes Since $\nabla \times \mathbf{F} = \mathbf{k}$,

$$\oint_{\partial S} \mathbf{F} \cdot d\mathbf{r} = \iint_S 1 \, dA = \pi(2^2 - 1^2) = 3\pi.$$

Step 3: Check components For a counterclockwise circle of radius R ,

$$\oint \mathbf{F} \cdot d\mathbf{r} = \pi R^2.$$

Thus the outer circle contributes 4π , while the clockwise inner circle contributes $-\pi$. Their sum is 3π .

Verification Assigning both circles counterclockwise would give 5π and violate the induced-boundary convention.