

Question 4

Let C be the intersection of the cylinder $x^2 + y^2 = 4$ with the plane $z = x + y$. Orient C consistently with the upward normal to the planar region it bounds. For

$$\mathbf{F} = \left\langle -\frac{y}{2}, \frac{x}{2}, 0 \right\rangle,$$

compute $\oint_C \mathbf{F} \cdot d\mathbf{r}$.

Tasks

1. Identify a convenient spanning surface and its vector element.
2. Compute the curl flux.
3. Explain why the tilt of the curve does not alter the result.

Question 4 – Solution

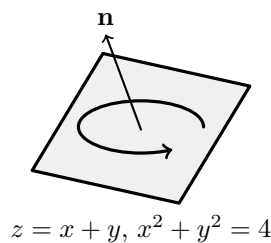
Strategy. Use the planar patch $z = x + y$ over the radius-2 disk; its vertical vector-area component is especially simple.

Step 1: Curl and vector area We have

$$\nabla \times \mathbf{F} = \langle 0, 0, 1 \rangle.$$

For $z = x + y$, the upward vector element is

$$\mathbf{n} dS = \langle -1, -1, 1 \rangle dA.$$



Step 2: Apply Stokes

$$\begin{aligned} \oint_C \mathbf{F} \cdot d\mathbf{r} &= \iint_{x^2+y^2 \leq 4} \langle 0, 0, 1 \rangle \cdot \langle -1, -1, 1 \rangle dA \\ &= \iint_{x^2+y^2 \leq 4} 1 dA = \boxed{4\pi}. \end{aligned}$$

Step 3: Interpret The curl is vertical, so only the vertical component of the oriented area vector matters. That component equals the area element of the horizontal projection, whose area is 4π .

Verification Reversing the curve would force the downward normal and change the answer to -4π .