

**Question 3**

Let  $C$  be the equator of the upper hemisphere  $x^2 + y^2 + z^2 = 9$ , oriented as the boundary of the outward-oriented hemisphere. For

$$\mathbf{F} = \langle -y, x, z \rangle,$$

find  $\oint_C \mathbf{F} \cdot d\mathbf{r}$  without parametrizing the hemisphere.

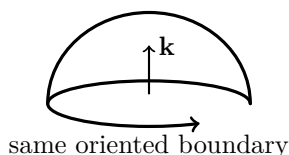
**Tasks**

1. Determine the induced direction of  $C$  as viewed from above.
2. Replace the hemisphere by a simpler spanning surface.
3. Evaluate the circulation and explain why the replacement is valid.

### Question 3 – Solution

**Strategy.** Stokes' Theorem permits any smooth oriented spanning surface with the same oriented boundary; use the equatorial disk.

**Step 1: Boundary orientation** The outward normal on the upper hemisphere induces counterclockwise motion around the equator when viewed from  $+z$ .



**Step 2: Choose the disk** Let  $D$  be  $x^2 + y^2 \leq 9$ ,  $z = 0$ , with upward normal. It has the same oriented boundary as the hemisphere. Since

$$\nabla \times \mathbf{F} = \langle 0, 0, 2 \rangle,$$

Stokes' Theorem gives

$$\oint_C \mathbf{F} \cdot d\mathbf{r} = \iint_D 2 \, dA = 2(9\pi) = 18\pi.$$

**Step 3: Explain the replacement** Both the hemisphere and disk are orientable, share  $C$ , and lie where  $\mathbf{F}$  has continuous derivatives. Each curl flux therefore equals the same boundary circulation.

**Verification** On the equator,  $\mathbf{F} = \langle -y, x, 0 \rangle$  is tangent in the counterclockwise direction and has magnitude 3; multiplying by the circumference  $6\pi$  also gives  $18\pi$ .