

Question 1

Let C be the unit circle $x^2 + y^2 = 1$ in the plane $z = 0$, oriented counterclockwise when viewed from above. For

$$\mathbf{F}(x, y, z) = \langle -y, x, z \rangle,$$

evaluate $\oint_C \mathbf{F} \cdot d\mathbf{r}$ using Stokes' Theorem.

Tasks

1. Choose an oriented spanning surface and compute $\nabla \times \mathbf{F}$.
2. Apply Stokes' Theorem.
3. Verify the answer by direct parametrization of C .

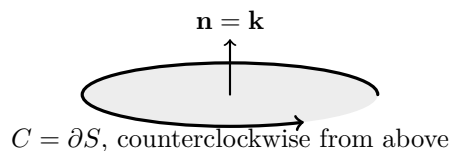
Question 1 – Solution

Strategy. Span the circle by its flat disk; the prescribed counterclockwise direction selects the upward normal.

Step 1: Curl and orientation

$$\nabla \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \partial_x & \partial_y & \partial_z \\ -y & x & z \end{vmatrix} = \langle 0, 0, 2 \rangle.$$

Let S be the unit disk with $\mathbf{n} = \mathbf{k}$.



Step 2: Apply Stokes

$$\oint_C \mathbf{F} \cdot d\mathbf{r} = \iint_S (\nabla \times \mathbf{F}) \cdot \mathbf{n} \, dS = \iint_S 2 \, dA = 2\pi.$$

Step 3: Direct check With $\mathbf{r}(t) = \langle \cos t, \sin t, 0 \rangle$,

$$\mathbf{F}(\mathbf{r}(t)) = \langle -\sin t, \cos t, 0 \rangle = \mathbf{r}'(t).$$

Therefore $\int_0^{2\pi} \mathbf{F} \cdot \mathbf{r}' \, dt = \int_0^{2\pi} 1 \, dt = 2\pi$.

Verification The boundary is counterclockwise from the tip of $\mathbf{k}$, so the orientation pairing is correct.