

Question 10

Let S be the closed cylinder $x^2 + y^2 = 1$, $0 \leq z \leq 2$, including its top and bottom disks, with outward orientation. For

$$\mathbf{F}(x, y, z) = \langle x, y, -2z \rangle,$$

compute the net outward flux directly, without using a general flux theorem.

Tasks

1. Compute the flux through the curved side.
2. Compute the flux through each cap with the correct outward normal.
3. Sum the contributions and interpret the cancellation.

Question 10 – Solution

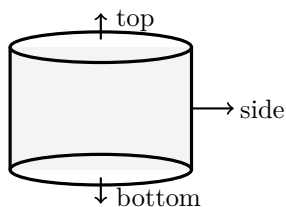
Strategy. Decompose the closed surface into three oriented pieces and track the outward normal on each one.

Step 1: Curved side Parametrize by

$$\mathbf{r}(\theta, z) = \langle \cos \theta, \sin \theta, z \rangle.$$

The outward vector element is $\langle \cos \theta, \sin \theta, 0 \rangle d\theta dz$. Its dot product with $\mathbf{F}$ is 1, so

$$\Phi_{\text{side}} = \int_0^{2\pi} \int_0^2 1 \, dz \, d\theta = 4\pi.$$



Step 2: Caps On the top disk $z = 2$, the outward normal is $\mathbf{k}$, and

$$\mathbf{F} \cdot \mathbf{k} = -2z = -4.$$

Thus $\Phi_{\text{top}} = -4\pi$. On the bottom disk $z = 0$, the outward normal is $-\mathbf{k}$, but the vertical field component is zero, so $\Phi_{\text{bottom}} = 0$.

Step 3: Net flux

$$\boxed{\Phi_{\text{net}} = 4\pi - 4\pi + 0 = 0}.$$

Verification The side carries 4π units outward while the top carries the same amount inward; the signed balance is zero even though neither contribution vanishes.