

**Question 6**

Let  $E$  be the tetrahedron in the first octant bounded by the coordinate planes and  $x + y + z = 1$ . Let  $S = \partial E$  carry the outward orientation, and let

$$\mathbf{F}(x, y, z) = \langle x, y, z \rangle.$$

Compute the total outward flux directly by summing the four faces.

**Tasks**

1. Determine the flux through the three coordinate-plane faces.
2. Parametrize the slanted face with its outward orientation.
3. Sum the face contributions and verify the sign.

## Question 6 – Solution

**Strategy.** The field has zero normal component on each coordinate face, leaving only the slanted triangular face.

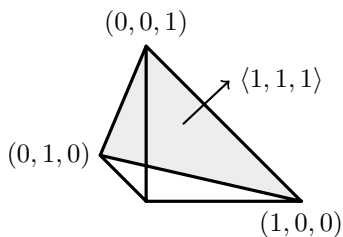
**Step 1: Coordinate faces** On  $x = 0$ , the outward normal is  $-\mathbf{i}$  and  $\mathbf{F} \cdot (-\mathbf{i}) = -x = 0$ . Similarly, the flux densities on  $y = 0$  and  $z = 0$  vanish. These three fluxes are all zero.

**Step 2: Slanted face** Write  $z = 1 - x - y$  over

$$D = \{(x, y) : x \geq 0, y \geq 0, x + y \leq 1\}.$$

The outward side points away from the origin, and its vector element is

$$\mathbf{n} dS = \langle 1, 1, 1 \rangle dx dy.$$



On this face,

$$\mathbf{F} \cdot \langle 1, 1, 1 \rangle = x + y + z = 1.$$

**Step 3: Integrate and sum**

$$\iint_{\partial E} \mathbf{F} \cdot \mathbf{n} dS = 0 + \iint_D 1 dA = \boxed{\frac{1}{2}}.$$

**Verification** The field points away from the origin and is tangent to the coordinate faces, so all nonzero flux must leave through the slanted face and must be positive.