

Question 5

Let S be the lateral cone

$$z = \sqrt{x^2 + y^2}, \quad 0 \leq z \leq 2,$$

oriented away from the z -axis. For $\mathbf{F} = \langle 0, 0, z \rangle$, compute the flux across S .

Tasks

1. Parametrize the cone and choose the requested orientation.
2. Evaluate the oriented surface integral.
3. Address the singular parameter point at the cone's vertex.

Question 5 – Solution

Strategy. The normal pointing horizontally away from the axis also has a downward vertical component, which determines the sign of the flux.

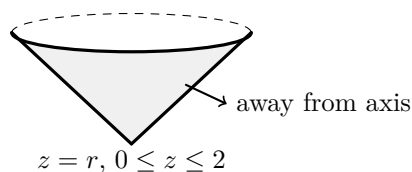
Step 1: Oriented cross product Use

$$\mathbf{r}(u, v) = \langle u \cos v, u \sin v, u \rangle, \quad 0 \leq u \leq 2, \quad 0 \leq v \leq 2\pi.$$

The ordered product

$$\mathbf{r}_v \times \mathbf{r}_u = \langle u \cos v, u \sin v, -u \rangle$$

has a radial component pointing away from the axis.



Step 2: Integrate On the cone, $\mathbf{F} = \langle 0, 0, u \rangle$, so

$$\mathbf{F} \cdot (\mathbf{r}_v \times \mathbf{r}_u) = -u^2.$$

Therefore

$$\begin{aligned} \iint_S \mathbf{F} \cdot \mathbf{n} \, dS &= \int_0^{2\pi} \int_0^2 -u^2 \, du \, dv \\ &= \boxed{-\frac{16\pi}{3}}. \end{aligned}$$

Step 3: Vertex The parametrization is singular at $u = 0$, but that single surface point has zero area. Equivalently, integrate over $\varepsilon \leq u \leq 2$ and let $\varepsilon \rightarrow 0^+$.

Verification The chosen normal has negative z -component while the field points upward, so a negative flux is required.