

Question 4

For $a > 0$, let S_a be the sphere $x^2 + y^2 + z^2 = a^2$, oriented outward. Consider the inverse-square radial field

$$\mathbf{F}(x, y, z) = \frac{\langle x, y, z \rangle}{(x^2 + y^2 + z^2)^{3/2}}.$$

Tasks

1. Find $\mathbf{F} \cdot \mathbf{n}$ on S_a .
2. Compute the outward flux directly.
3. Explain why the result is independent of a .

Question 4 – Solution

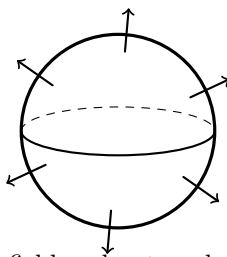
Strategy. On a centered sphere, the field and unit normal are parallel; only their magnitudes need to be compared.

Step 1: Normal component Write $\mathbf{x} = \langle x, y, z \rangle$. On S_a ,

$$\mathbf{n} = \frac{\mathbf{x}}{a}, \quad \mathbf{F} = \frac{\mathbf{x}}{a^3}.$$

Consequently,

$$\mathbf{F} \cdot \mathbf{n} = \frac{\mathbf{x} \cdot \mathbf{x}}{a^4} = \frac{1}{a^2}.$$



radial field and outward normal

Step 2: Integrate Since the sphere has area $4\pi a^2$,

$$\iint_{S_a} \mathbf{F} \cdot \mathbf{n} \, dS = \frac{1}{a^2} (4\pi a^2) = 4\pi.$$

Step 3: Interpret the scale Increasing the radius weakens the normal component by the factor a^{-2} while increasing spherical area by a^2 . These factors cancel exactly.

Verification The field is everywhere outward on S_a , so the positive sign is correct. The calculation is direct and uses no theorem about closed surfaces.