

Question 3

Let S be only the curved side of the cylinder

$$x^2 + y^2 = 4, \quad 0 \leq z \leq 3,$$

oriented away from the z -axis. For $\mathbf{F} = \langle x, y, z^2 \rangle$, find the outward flux.

Tasks

1. Parametrize the curved side with the correct orientation.
2. Compute the oriented vector area element.
3. Evaluate and interpret the flux.

Question 3 – Solution

Strategy. Use angle and height parameters. The outward cross product is horizontal, so the vertical component of the field contributes nothing.

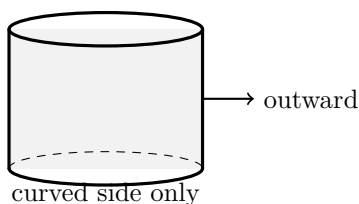
Step 1: Parametrize

$$\mathbf{r}(\theta, z) = \langle 2 \cos \theta, 2 \sin \theta, z \rangle, \quad 0 \leq \theta \leq 2\pi, \quad 0 \leq z \leq 3.$$

The ordered cross product

$$\mathbf{r}_\theta \times \mathbf{r}_z = \langle 2 \cos \theta, 2 \sin \theta, 0 \rangle$$

points away from the axis.



Step 2: Form the integrand On S ,

$$\mathbf{F} = \langle 2 \cos \theta, 2 \sin \theta, z^2 \rangle,$$

and therefore

$$\mathbf{F} \cdot (\mathbf{r}_\theta \times \mathbf{r}_z) = 4 \cos^2 \theta + 4 \sin^2 \theta = 4.$$

Step 3: Integrate

$$\iint_S \mathbf{F} \cdot \mathbf{n} \, dS = \int_0^{2\pi} \int_0^3 4 \, dz \, d\theta = 24\pi.$$

Verification The flux density is constantly positive, so the field points outward through every point of the curved side. The z^2 component is tangent to that side and correctly disappears.