

Question 1

Let S be the part of the plane $z = 1 + x + y$ above the unit square $0 \leq x, y \leq 1$, oriented upward. Let

$$\mathbf{F}(x, y, z) = \langle x, y, z \rangle.$$

Tasks

1. Find the upward-oriented vector surface element.
2. Compute the flux $\iint_S \mathbf{F} \cdot \mathbf{n} \, dS$.
3. State how the result changes under downward orientation.

Question 1 – Solution

Strategy. Parametrize the graph by (x, y) and retain the cross product itself, not merely its magnitude.

Step 1: Oriented element Use

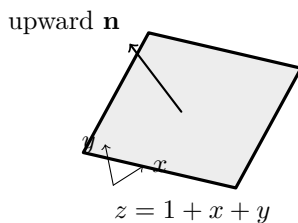
$$\mathbf{r}(x, y) = \langle x, y, 1 + x + y \rangle.$$

Then

$$\mathbf{r}_x = \langle 1, 0, 1 \rangle, \quad \mathbf{r}_y = \langle 0, 1, 1 \rangle,$$

so the upward vector area element is

$$\mathbf{n} \, dS = (\mathbf{r}_x \times \mathbf{r}_y) \, dx \, dy = \langle -1, -1, 1 \rangle \, dx \, dy.$$



Step 2: Dot product On the surface, $z = 1 + x + y$, and hence

$$\mathbf{F} \cdot (\mathbf{r}_x \times \mathbf{r}_y) = -x - y + z = 1.$$

Therefore

$$\iint_S \mathbf{F} \cdot \mathbf{n} \, dS = \int_0^1 \int_0^1 1 \, dx \, dy = 1.$$

Step 3: Reverse orientation A downward normal is the negative of the chosen one, so the downward flux is $\boxed{-1}$.

Verification The cross product has positive third component, confirming the upward orientation; changing only the orientation must negate a flux integral.