

Question 10

The same planar patch is described over the unit square by

$$\mathbf{r}(u, v) = \langle u, v, u + 2v \rangle, \quad \mathbf{s}(p, q) = \langle 1 - p, q, 1 - p + 2q \rangle.$$

Tasks

1. Show that the parametrizations have the same image but opposite orientations.
2. Evaluate $\iint_S (x + y) \, dS$ using each parametrization.
3. Explain why reversing orientation cannot change a scalar surface integral.

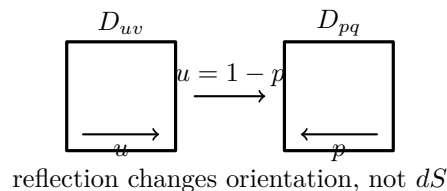
Question 10 – Solution

Strategy. Recognize $\mathbf{s}(p, q) = \mathbf{r}(1 - p, q)$, then compare cross products by magnitude rather than sign.

Step 1: Image and orientation The reflection $(u, v) = (1 - p, q)$ maps the unit square onto itself, so the images agree. Moreover,

$$\mathbf{r}_u \times \mathbf{r}_v = \langle -1, -2, 1 \rangle, \quad \mathbf{s}_p \times \mathbf{s}_q = \langle 1, 2, -1 \rangle.$$

The normals are negatives, so the orientations are opposite; both magnitudes are $\sqrt{6}$.



Step 2: First parametrization

$$\iint_S (x + y) dS = \int_0^1 \int_0^1 (u + v) \sqrt{6} du dv = \boxed{\sqrt{6}}.$$

Step 3: Second parametrization

$$\iint_S (x + y) dS = \int_0^1 \int_0^1 (1 - p + q) \sqrt{6} dp dq = \boxed{\sqrt{6}}.$$

Verification A scalar surface integral uses $\|\mathbf{r}_u \times \mathbf{r}_v\|$, so reversing the cross product does not change dS . Here the reflection has Jacobian determinant -1 , whose absolute value is 1, confirming the equality under change of parameters.