

Question 7

Let S be the entire boundary of the unit cube $0 \leq x, y, z \leq 1$.

Tasks

1. Decompose S into its six planar faces.
2. Evaluate $\iint_S (x + y + z) dS$.
3. Find the average value of $x + y + z$ on the cube's boundary and verify it by symmetry.

Question 7 – Solution

Strategy. Pair opposite faces. Each pair produces the same contribution, avoiding six unrelated computations.

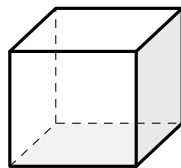
Step 1: Pair the x -faces On $x = 0$, $dS = dy dz$ and

$$\int_0^1 \int_0^1 (y + z) dy dz = 1.$$

On $x = 1$,

$$\int_0^1 \int_0^1 (1 + y + z) dy dz = 2.$$

Thus the pair perpendicular to the x -axis contributes 3.



six faces of the unit cube

Step 2: Use coordinate symmetry The cube and integrand are unchanged by permuting x, y, z . Consequently, the two y -faces and two z -faces also contribute 3 apiece. Hence

$$\boxed{\iint_S (x + y + z) dS = 3 + 3 + 3 = 9}.$$

Step 3: Average The six unit-square faces give $\text{area}(S) = 6$, so

$$\boxed{\text{Avg}_S(x + y + z) = \frac{9}{6} = \frac{3}{2}}.$$

Verification The map $(x, y, z) \mapsto (1 - x, 1 - y, 1 - z)$ preserves the boundary and pairs values whose sum is 3. Therefore their average is $3/2$, confirming the computation.