

Question 6

Let S be the upper hemisphere

$$z = \sqrt{9 - x^2 - y^2}, \quad z \geq 0.$$

Evaluate $\iint_S z \, dS$ by viewing S as a graph, and find the average value of z on S .

Tasks

1. Derive the graph surface element and address its boundary singularity.
2. Explain the cancellation that simplifies the integrand.
3. Evaluate the integral and average value.

Question 6 – Solution

Strategy. The graph factor becomes unbounded at the equator, but multiplication by z cancels it exactly in the interior.

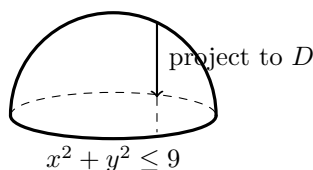
Step 1: Graph factor Let $g(x, y) = \sqrt{9 - x^2 - y^2}$ over the disk $D : x^2 + y^2 \leq 9$. Since

$$g_x = -\frac{x}{z}, \quad g_y = -\frac{y}{z},$$

we have, for $z > 0$,

$$\sqrt{1 + g_x^2 + g_y^2} = \sqrt{1 + \frac{x^2 + y^2}{z^2}} = \frac{3}{z}.$$

Thus $dS = (3/z)dA$ away from the boundary.



Step 2: Cancel and integrate The weighted element is simply

$$z dS = z \frac{3}{z} dA = 3 dA.$$

Therefore

$$\boxed{\iint_S z dS = 3 \text{ area}(D) = 27\pi.}$$

The hemisphere has area $2\pi(3^2) = 18\pi$, so

$$\boxed{\text{Avg}_S(z) = \frac{27\pi}{18\pi} = \frac{3}{2}.}$$

Verification Integrating over disks of radius $R < 3$ gives $3\pi R^2$; taking $R \rightarrow 3^-$ justifies the boundary limit and yields 27π .