

Question 1

Let S be the portion of the plane $z = 2x + y$ above the rectangle

$$R = \{(x, y) : 0 \leq x \leq 1, 0 \leq y \leq 2\}.$$

Tasks

1. Find the scalar surface element dS .
2. Compute the area of S .
3. Evaluate $\iint_S (x + z) dS$ and find the average value of $x + z$ on S .

Question 1 – Solution

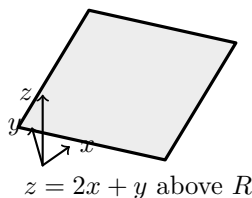
Strategy. Treat the plane as a graph, so the same constant area factor converts every integral over S into an integral over R .

Step 1: Surface element For $z = g(x, y) = 2x + y$,

$$g_x = 2, \quad g_y = 1,$$

and therefore

$$dS = \sqrt{1 + g_x^2 + g_y^2} dA = \sqrt{6} dA.$$



Step 2: Area

$$\text{area}(S) = \iint_R \sqrt{6} dA = \sqrt{6}(1)(2) = \boxed{2\sqrt{6}}.$$

Step 3: Weighted integral On S , $x + z = 3x + y$. Hence

$$\begin{aligned} \iint_S (x + z) dS &= \sqrt{6} \int_0^1 \int_0^2 (3x + y) dy dx \\ &= \sqrt{6} \int_0^1 (6x + 2) dx = \boxed{5\sqrt{6}}. \end{aligned}$$

The average value is

$$\frac{1}{\text{area}(S)} \iint_S (x + z) dS = \frac{5}{2}.$$

Verification The average $5/2$ lies between the minimum 0 and maximum 5 of $3x + y$ on R .