

Question 9

Let

$$\mathbf{r}(u, v) = \langle u \cos v, u \sin v, u^2 \rangle, \quad 1 \leq u \leq 2, \quad 0 \leq v \leq \pi.$$

Describe all boundary curves of the surface patch.

Tasks

1. Find the curves corresponding to $u = 1$ and $u = 2$.
2. Find the curves corresponding to $v = 0$ and $v = \pi$.
3. Identify the surface and verify that the four curves join correctly.

Question 9 – Solution

Strategy. Each edge of the rectangular parameter domain maps to one boundary curve.

Step 1: Constant- u edges

$$u = 1 : \langle \cos v, \sin v, 1 \rangle, \quad u = 2 : \langle 2 \cos v, 2 \sin v, 4 \rangle, \quad 0 \leq v \leq \pi.$$

These are semicircles at heights 1 and 4.

Step 2: Constant- v edges

$$v = 0 : \langle u, 0, u^2 \rangle, \quad v = \pi : \langle -u, 0, u^2 \rangle, \quad 1 \leq u \leq 2.$$

These are parabolic side curves in the plane $y = 0$.



half-annular paraboloid patch

Step 3: Identify and join Since $x^2 + y^2 = u^2$ and $z = u^2$, the patch lies on $z = x^2 + y^2$ above the half-annulus $1 \leq x^2 + y^2 \leq 4$, $y \geq 0$.

Verification The four corner images are $(1, 0, 1)$, $(-1, 0, 1)$, $(2, 0, 4)$, and $(-2, 0, 4)$; each occurs as an endpoint of exactly two adjacent boundary curves.