

Question 7

Consider

$$\mathbf{r}(u, v) = \langle u^2, v, u^3 \rangle, \quad -1 \leq u \leq 1, \quad -1 \leq v \leq 1.$$

Tasks

1. Eliminate u to find an implicit relation between x and z .
2. Determine where the parametrization is regular.
3. Explain the geometric significance of the failure set.

Question 7 – Solution

Strategy. Compare $x = u^2$ and $z = u^3$, then locate zeros of the tangent cross product.

Step 1: Eliminate the parameter

$$z^2 = u^6 = (u^2)^3 = x^3, \quad x \geq 0.$$

The image is the patch of the cuspidal cylinder $z^2 = x^3$ with $0 \leq x \leq 1$ and $-1 \leq y \leq 1$.

Step 2: Test regularity

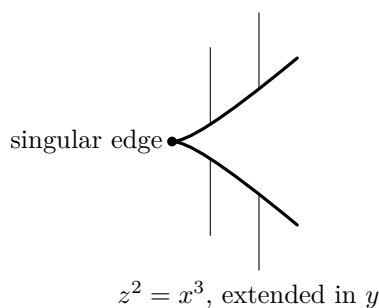
$$\mathbf{r}_u = \langle 2u, 0, 3u^2 \rangle, \quad \mathbf{r}_v = \langle 0, 1, 0 \rangle,$$

so

$$\mathbf{r}_u \times \mathbf{r}_v = \langle -3u^2, 0, 2u \rangle.$$

This vector vanishes exactly when $u = 0$. Therefore the parametrization is regular for $u \neq 0$ and singular along

$$\boxed{\mathbf{r}(0, v) = \langle 0, v, 0 \rangle}.$$



Step 3: Interpret That line is the sharp cuspidal edge. At it, the u -direction tangent collapses to zero, so the two tangent vectors cannot span a tangent plane.

Verification For $u \neq 0$, the third component $2u$ of the cross product is nonzero, proving regularity away from the edge.