

Question 6

Analyze the parametrized surface

$$\mathbf{r}(u, v) = \langle u \cos v, u \sin v, v \rangle, \quad 0 \leq u \leq 2, \quad 0 \leq v \leq 2\pi.$$

Tasks

1. Identify the surface geometrically.
2. Describe the coordinate curves $u = \text{constant}$ and $v = \text{constant}$.
3. Test regularity on the entire parameter domain.

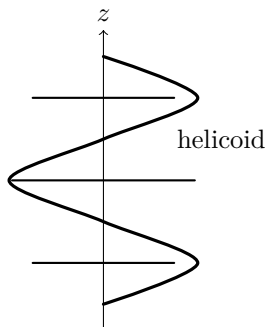
Question 6 – Solution

Strategy. Interpret radius, angle, and height directly, then test the tangent cross product.

Step 1: Identify In cylindrical terms,

$$\rho = u, \quad \theta = v, \quad z = v.$$

As the angle increases, height increases at the same rate; the image is a helicoid patch.



Step 2: Coordinate curves If $u = u_0$, then

$$\langle u_0 \cos v, u_0 \sin v, v \rangle$$

is a helix of radius u_0 ; for $u_0 = 0$ it is the z -axis. If $v = v_0$, varying u traces a radial line segment at height $z = v_0$.

Step 3: Regularity

$$\mathbf{r}_u = \langle \cos v, \sin v, 0 \rangle, \quad \mathbf{r}_v = \langle -u \sin v, u \cos v, 1 \rangle,$$

and

$$\boxed{\mathbf{r}_u \times \mathbf{r}_v = \langle \sin v, -\cos v, u \rangle}.$$

Its magnitude is $\sqrt{1 + u^2} > 0$, even at $u = 0$, so the parametrization is regular everywhere.

Verification The first two components of the cross product have squared sum 1, preventing the normal from vanishing.