

Question 5

Parametrize the conical frustum

$$z = \sqrt{x^2 + y^2}, \quad 1 \leq z \leq 2,$$

and find a normal pointing away from the z -axis.

Tasks

1. Choose radial and angular parameters with correct bounds.
2. Compute both possible normal orientations.
3. Select and verify the outward normal.

Question 5 – Solution

Strategy. On this cone the radius equals the height, so one parameter can serve both roles.

Step 1: Parametrize

$$\mathbf{r}(u, v) = \langle u \cos v, u \sin v, u \rangle, \quad 1 \leq u \leq 2, \quad 0 \leq v \leq 2\pi.$$

Step 2: Cross the tangents

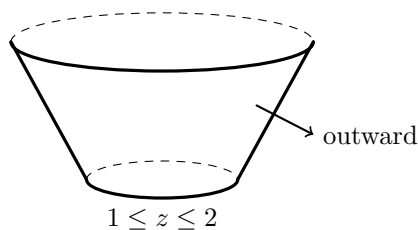
$$\mathbf{r}_u = \langle \cos v, \sin v, 1 \rangle, \quad \mathbf{r}_v = \langle -u \sin v, u \cos v, 0 \rangle.$$

Thus

$$\mathbf{r}_u \times \mathbf{r}_v = \langle -u \cos v, -u \sin v, u \rangle.$$

This points inward horizontally. Reverse the order to obtain

$$\boxed{\mathbf{r}_v \times \mathbf{r}_u = \langle u \cos v, u \sin v, -u \rangle}.$$



Step 3: Verify Its horizontal component is $\langle x, y, 0 \rangle$, pointing away from the axis. It also agrees with the outward direction of $x^2 + y^2 - z^2 = 0$, whose gradient is $2\langle x, y, -z \rangle$.