

Question 3

Parametrize the half-cylinder

$$x^2 + y^2 = 4, \quad y \geq 0, \quad -1 \leq z \leq 3,$$

and choose the orientation whose normal points outward.

Tasks

1. Give a parameter domain that covers the patch once, apart from boundary seams.
2. Compute a normal from the tangent vectors.
3. Verify that the normal is outward.

Question 3 – Solution

Strategy. Use an angular parameter for the circular direction and a height parameter for z .

Step 1: Parametrize

$$\mathbf{r}(\theta, s) = \langle 2 \cos \theta, 2 \sin \theta, s \rangle, \quad 0 \leq \theta \leq \pi, \quad -1 \leq s \leq 3.$$

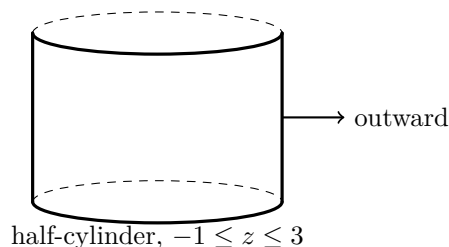
The interval $0 \leq \theta \leq \pi$ enforces $y \geq 0$.

Step 2: Tangent vectors

$$\mathbf{r}_\theta = \langle -2 \sin \theta, 2 \cos \theta, 0 \rangle, \quad \mathbf{r}_s = \langle 0, 0, 1 \rangle.$$

Their ordered cross product is

$$\mathbf{r}_\theta \times \mathbf{r}_s = \langle 2 \cos \theta, 2 \sin \theta, 0 \rangle.$$



Step 3: Check orientation At each point, the horizontal radial vector from the z -axis is $\langle x, y, 0 \rangle = \langle 2 \cos \theta, 2 \sin \theta, 0 \rangle$, exactly the cross product. Hence it points outward.

Verification Its magnitude is 2, never zero, so the parametrization is regular everywhere on the patch, including its boundary curves.