

Question 2

Parametrize the paraboloid $z = x^2 + y^2$ as a graph. At the point $P = (1, -1, 2)$, find a normal vector and the tangent plane.

Tasks

1. Give a two-parameter graph parametrization.
2. Compute the tangent vectors and their cross product at P .
3. Write and verify the tangent-plane equation.

Question 2 – Solution

Strategy. Use $x = u$, $y = v$, and let the defining graph supply the third component.

Step 1: Parametrize

$$\mathbf{r}(u, v) = \langle u, v, u^2 + v^2 \rangle, \quad (u, v) \in \mathbb{R}^2.$$

The point P corresponds to $(u, v) = (1, -1)$.

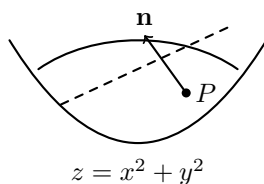
Step 2: Normal vector

$$\mathbf{r}_u = \langle 1, 0, 2u \rangle, \quad \mathbf{r}_v = \langle 0, 1, 2v \rangle,$$

so

$$\mathbf{r}_u \times \mathbf{r}_v = \langle -2u, -2v, 1 \rangle.$$

At P , an upward normal is $\boxed{\langle -2, 2, 1 \rangle}$.



Step 3: Tangent plane

$$-2(x - 1) + 2(y + 1) + (z - 2) = 0,$$

or

$$\boxed{z = 2x - 2y - 2}.$$

Verification Substitution of $(1, -1, 2)$ gives $2 = 2 + 2 - 2$. The normal from the graph equation $x^2 + y^2 - z = 0$ is $\langle 2, -2, -1 \rangle$, the negative of the chosen normal.