

Question 1

Consider the surface patch

$$\mathbf{r}(u, v) = \langle u, v, 1 + 2u - v \rangle, \quad 0 \leq u \leq 2, \quad 0 \leq v \leq 1.$$

Tasks

1. Identify the surface and its parameter-domain image.
2. Find two tangent vectors and a normal vector.
3. List the four corner points of the patch.

Question 1 – Solution

Strategy. Read $x = u$ and $y = v$, eliminate the parameters, and map the rectangle's four corners.

Step 1: Identify the surface Since $x = u$ and $y = v$,

$$z = 1 + 2x - y.$$

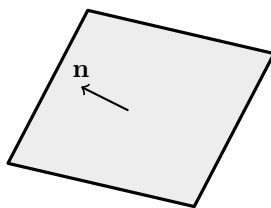
Thus the image is the portion of this plane above the rectangle $0 \leq x \leq 2$, $0 \leq y \leq 1$.

Step 2: Tangent and normal vectors

$$\mathbf{r}_u = \langle 1, 0, 2 \rangle, \quad \mathbf{r}_v = \langle 0, 1, -1 \rangle.$$

Therefore

$$\boxed{\mathbf{r}_u \times \mathbf{r}_v = \langle -2, 1, 1 \rangle}.$$



planar parameter rectangle

Step 3: Map the corners

$$\mathbf{r}(0, 0) = (0, 0, 1),$$

$$\mathbf{r}(2, 0) = (2, 0, 5),$$

$$\mathbf{r}(0, 1) = (0, 1, 0),$$

$$\mathbf{r}(2, 1) = (2, 1, 4).$$

These four points bound the parallelogram patch.

Verification The normal $\langle -2, 1, 1 \rangle$ is perpendicular to both tangent vectors, and each corner satisfies $z = 1 + 2x - y$.