

Question 10

Let

$$\mathbf{F}(x, y, z) = \langle -yz, xz, xy \rangle.$$

Tasks

1. Compute the divergence and curl everywhere.
2. Evaluate the curl at $P = (1, 2, 3)$ and give its direction.
3. Find every point at which the curl vanishes.

Question 10 – Solution

Strategy. Compute the global formulas first; then solve the component equations of the curl simultaneously.

Step 1: Divergence Each component omits its matching differentiation variable, so

$$\nabla \cdot \mathbf{F} = \frac{\partial(-yz)}{\partial x} + \frac{\partial(xz)}{\partial y} + \frac{\partial(xy)}{\partial z} = \boxed{0}.$$

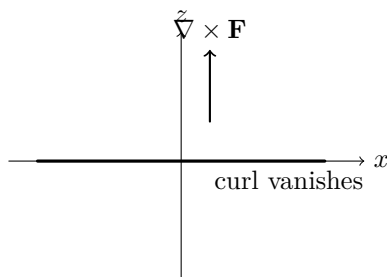
Step 2: Curl

$$\begin{aligned}\nabla \times \mathbf{F} &= \langle R_y - Q_z, P_z - R_x, Q_x - P_y \rangle \\ &= \langle x - x, -y - y, z - (-z) \rangle \\ &= \boxed{\langle 0, -2y, 2z \rangle}.\end{aligned}$$

At $P = (1, 2, 3)$,

$$(\nabla \times \mathbf{F})(P) = \boxed{\langle 0, -4, 6 \rangle},$$

which points in the direction $\boxed{\langle 0, -2, 3 \rangle / \sqrt{13}}$.



Step 3: Vanishing locus The curl is zero exactly when

$$-2y = 0, \quad 2z = 0.$$

Thus $y = z = 0$ while x is arbitrary:

$$\boxed{\nabla \times \mathbf{F} = \mathbf{0} \text{ exactly on the } x\text{-axis}.}$$

Verification Substituting $(x, 0, 0)$ into the curl formula gives $\mathbf{0}$, and any point off the x -axis has $y \neq 0$ or $z \neq 0$, making at least one curl component nonzero.