

Question 8

A linear planar field has the expansion-rotation form

$$\mathbf{F}(x, y) = \langle ax - by, bx + ay \rangle.$$

Its divergence is 4 and its scalar curl is 6 everywhere.

Tasks

1. Determine a and b .
2. Write the resulting field explicitly.
3. Separate it into pure expansion and pure rotation parts.

Question 8 – Solution

Strategy. Divergence isolates the expansion coefficient, while scalar curl isolates twice the rotation coefficient.

Step 1: Compute the invariants

$$\nabla \cdot \mathbf{F} = a + a = 2a, \quad \text{curl}_{2D} \mathbf{F} = b - (-b) = 2b.$$

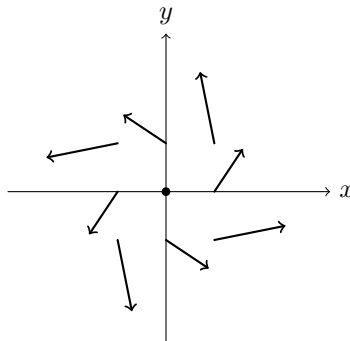
Thus

$$2a = 4, \quad 2b = 6,$$

so $\boxed{a = 2}$ and $\boxed{b = 3}$.

Step 2: Write the field

$$\boxed{\mathbf{F}(x, y) = \langle 2x - 3y, 3x + 2y \rangle}.$$



outward expansion plus counterclockwise rotation

Step 3: Decompose

$$\mathbf{F} = \underbrace{\langle 2x, 2y \rangle}_{\text{pure expansion}} + \underbrace{\langle -3y, 3x \rangle}_{\text{counterclockwise rotation}}.$$

The first part has divergence 4 and zero curl; the second has zero divergence and scalar curl 6.

Verification Recombining the two displayed parts returns both components of the boxed field and reproduces the prescribed invariants.