

Question 3

Let

$$\mathbf{F}(x, y, z) = \langle -2y, 2x, z^2 \rangle.$$

At $P = (1, -1, 2)$, determine the local expansion rate and the curl vector.

Tasks

1. Compute and evaluate the divergence.
2. Compute and evaluate the curl.
3. Interpret the curl's axis, direction, and infinitesimal angular-speed relation.

Question 3 – Solution

Strategy. Use divergence for local volume change and curl for twice the infinitesimal angular-velocity vector.

Step 1: Expansion rate

$$\nabla \cdot \mathbf{F} = 0 + 0 + 2z = 2z.$$

At P , this is

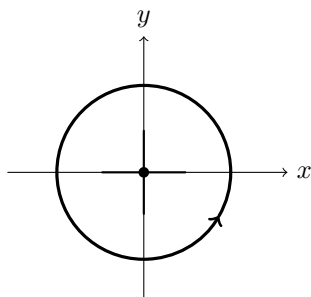
$$\boxed{(\nabla \cdot \mathbf{F})(P) = 4},$$

so the field has positive local expansion there.

Step 2: Curl

$$\nabla \times \mathbf{F} = \langle 0 - 0, 0 - 0, 2 - (-2) \rangle = \boxed{\langle 0, 0, 4 \rangle}.$$

This curl is constant, so its value at P is the same vector.



$\nabla \times \mathbf{F}$ points along $+z$

Step 3: Interpret The curl points along the positive z -axis, indicating counterclockwise rotation in horizontal planes when viewed from $+z$. The associated infinitesimal angular-velocity vector is half the curl:

$$\boxed{\vec{\omega} = \langle 0, 0, 2 \rangle}.$$

Verification The horizontal field $\langle -2y, 2x \rangle$ is rigid rotation with angular speed 2, which matches $|\nabla \times \mathbf{F}|/2 = 2$.