

Question 2

Consider the position field

$$\mathbf{F}(x, y, z) = \langle x, y, z \rangle.$$

Tasks

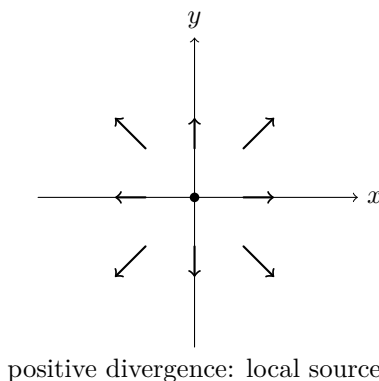
1. Compute its divergence and curl.
2. Interpret the divergence as local expansion or contraction.
3. Explain why the field has no local rotational tendency.

Question 2 – Solution

Strategy. Differentiate the matching coordinate in each divergence term and compare the cross derivatives for curl.

Step 1: Divergence

$$\nabla \cdot \mathbf{F} = \frac{\partial x}{\partial x} + \frac{\partial y}{\partial y} + \frac{\partial z}{\partial z} = 1 + 1 + 1 = \boxed{3}.$$



The positive constant divergence describes uniform local expansion: arrows point outward and grow with distance from the origin.

Step 2: Curl

Every cross derivative is zero, so

$$\nabla \times \mathbf{F} = \langle 0 - 0, 0 - 0, 0 - 0 \rangle = \boxed{\mathbf{0}}.$$

Step 3: Interpret Curl measures infinitesimal rotational tendency. This radial field expands directly away from the origin without circulating around any axis, consistent with zero curl.

Verification The field is $\nabla[(x^2 + y^2 + z^2)/2]$. A gradient field has zero curl where the second partial derivatives are continuous, confirming the calculation.